



Math 1110

Quiz 2

Name: \_\_\_\_\_

Section (circle) 004/005

**Problem 1.** Determine whether the following statements are (always) true or (at least sometimes) false, and circle your response. Please give a brief explanation (in complete sentences!) – a reason why it's true, or an example where it fails.

- (a) If  $f(x)$  is a one-to-one function and is never zero, then the function

$$h(x) = \frac{1}{f(x)}$$

TRUE FALSE

is also one-to-one.

We need to show that if there were two  $x$ -values  $x_1$  and  $x_2$  such that  $h(x_1) = h(x_2)$ , then  $x_1 = x_2$ . So we begin by assuming that there are  $x_1$  and  $x_2$  such that  $h(x_1) = h(x_2)$ . Then substituting,  $\frac{1}{f(x_1)} = \frac{1}{f(x_2)}$  and accordingly,  $f(x_1) = f(x_2)$ . Since  $f$  is a one-to-one function, this means that  $x_1 = x_2$ , and we conclude  $h$  is one-to-one.

- (b) If  $f(x)$  and  $g(x)$  are two functions, then

$$f \circ g(x) = g \circ f(x)$$

TRUE FALSE

whenever both sides of the equation are defined.

Consider  $f(x) = x^2$  and  $g(x) = x + 1$ . Then

$$f \circ g(x) = (x + 1)^2 = x^2 + 2x + 1$$

whereas

$$g \circ f(x) = x^2 + 1$$

Note  $f \circ g(1) = 4$ , which does not equal  $g \circ f(1) = 2$ . In particular,  $f \circ g(x) \neq g \circ f(x)$  as functions.