

TEACHING PHILOSOPHY

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I want my students to be a part of the mathematical process and to think that mathematics is something to be understood and done. I ask my students to work through the material with a lot of the same methods that I use when working through new mathematical ideas: to engage critically with and ask questions about reading and lectures, to reason analytically from pre-existing knowledge to new knowledge, to communicate the results of their inquiries clearly and carefully and to apply their knowledge of mathematics to new or unfamiliar problems.

One way that I help students to engage critically with the material is by helping them learn to read mathematical texts. I reassure them that reading math takes time and make suggestions for reading math texts: read with a pencil in hand, work out the example yourself and compare with the text, take reading notes. The goal is for students to begin to synthesize what they have read and absorb the facts, so that class time can be used to ask questions, and deepen understanding.

Further, I ask my students to come to class ready to talk about the text they have just read so that we can have a discussion about what the material means. By conducting class under the assumption that students have read the text, I can use scarce class time to engage students in discussions of the key ideas. This mode of conducting class is not unfamiliar to many students, as it is the way that most humanities courses operate. Some days, I might begin classes by asking them to think about a relevant computation, either from a previous class or from the reading to reinforce the importance of preparation for class. This works especially well on days when I am starting a new concept that builds on old ones. For example, I might start a first lecture on derivatives by asking students to compute a limit of the form $\lim_{h \rightarrow 0} \frac{f(x_0+h)-f(x_0)}{h}$ for some simple function, since the rest of the class will be about understanding the meaning of limits of this kind. Other days I might begin class by asking students to write down the main point of the day's lesson as they understand it from the reading, so that they begin to think of teasing out the main concepts and explaining them. Then, at the end of class, we can return and amend the main point as students understood it at the beginning of class with more understanding gained over the course of class.

I encourage critical thinking by asking conceptual questions at many stages of the learning process and facilitating discussions about the answers. One source of inspiration for such questions is the "Good Questions Project" from Cornell, which provides multiple choice and true or false questions to tease out conceptual understanding of concepts. Asking conceptual

questions every class helps students get in the habit of thinking of math as something to be understood and discussed, rather than a series of computations. I recently ran into a former student on campus, who told me how much she had enjoyed my calculus class (her first college math class) because had never known math was something she could understand before!

I ask my students to communicate what they have learned in several ways and to various audiences. I have heard gasps of surprise on the first day of class when I ask for math to be explained in English sentences. One chance that students get to refine their mathematical communication skills is in working in small groups in class. I ask students to work in small groups on a particularly hard problem, or to derive a new formula from an known one. This lets them practice talking informally to an audience that is inclined to be friendly while still forming new ideas. To emphasize written communication in words I give the occasional essay quiz, asking students to highlight the most important concepts in the course so far and their relationships to each other. The first time I was an instructor for a Calculus I course I gave this assignment as the last in-class quiz of the semester. This had the effect of both focusing my students' review and reassuring me that my students could identify the key concepts in the course, limits, integrals and derivatives, and explain their relationships using their definitions and the Fundamental Theorem of Calculus.

For most students, the scary part of many math classes is in the application of tools they already know to a new problem. This is one of the most important skills we teach, especially for non-mathematicians, and it is hard! That said, the only way to learn to apply pre-existing tools to new problems is to just do it, and learn by trial and error. Group activities that ask for applications of existing knowledge to new problems force students to do exactly that in an environment that seems friendly to them. I divide the students into groups and give each group a different example or application and ask them to report back. At the end of class, the groups present their work to the class. Students gain understanding from each other, while seeing more examples than they could have considered in depth in their own groups. They can ask each other questions and sometimes see a different approach than the one that I would have taken. This teaching device has been especially successful with related rates because by the time that most calculus courses cover related rates the students have already learned all of the tools to do any problem. On an informal mid-semester evaluation, I was asked for more days like the day we did related rates problems in groups after one very quick example done with the whole class.

Asking my students to read, to discuss and refine their understanding of concepts in class, to investigate new applications, and to communicate their ideas all come with one goal in mind: help foster the development of the careful reasoning and precise exposition that form the heart of the study of mathematics. I want my students to be able to carry these skills into their future endeavors, whether those endeavors are other math classes, science classes, humanities classes, or simply life as a more informed, more critically thinking citizen.