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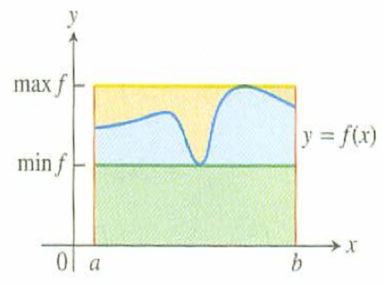
$$m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a).$$



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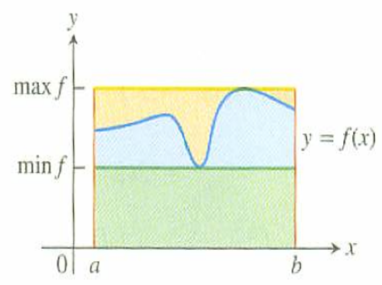

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$$m \leq f_{\text{avg}} \leq M$$

MVT for Integrals

Theorem 5.4.3 The Mean Value Theorem for Definite Integrals

If f is continuous on $[a, b]$, then at some point $c \in [a, b]$

$$f(c) = \frac{1}{b-a} \int_a^b f(x)\,dx.$$

That is, the average value of f is obtained at some point.

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This can also be written as

$$f(c)(b-a) = \int_a^b f(x)\,dx,$$

for some $c \in [a, b]$.

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Exercise. If f is continuous and $\int_1^3 f(x)\, dx = 8$, show that f takes on the value 4 at at least one point.

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MVT and FTC

FTC I has to do with $\frac{d}{dx}\int_a^x f(t)\,dt$.

Using the definition of a derivative and properties of integrals,

$$\frac{d}{dx}\int_a^x f(t)\,dt = \lim_{h\to 0}\frac{1}{h}\left(\int_a^{x+h} f(t)\,dt - \int_a^x f(t)\,dt\right) = \lim_{h\to 0}\frac{1}{h}\int_x^{x+h} f(t)\,dt.$$

What do you notice about the final expression?

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MVT for Derivatives

Recall:

Mean Value Theorem for Derivatives 4.2.4

If f is a continuous function on $[a, b]$, then there exists a $c \in [a, b]$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

This is a result about the average *rate of change* of f.

Namely, this says that the average rate of change of a continuous function is obtained at some point.

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FTC I-Theorem 5.5.4 Part I

If f is a continuous function on $[a, b]$, the $F(x) = \int_a^x f(t)\,dt$ is continuous on [a,b] and differentiable on (a,b). Furthermore, its derivative is:

$$F'(x) = \frac{d}{dx}\int_a^x f(t)\,dt = f(x), \text{ for all x in } [a, b].$$

FTC II-Theorem 5.4.4 Part II

If f is continuous at every point in $[a, b]$ and F is *any* antiderivative of f on $[a, b]$, then the value of the definite integral of f from a to b can be evaluated as follows

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Indefinite Integrals

The indefinite integral of f with respect to x is the family of all antiderivatives of f.

$$\int f(x)\,dx = F(x) + C,$$

where F is any antiderivative of f.

Example.

$$\int x^2\,dx = \frac{1}{3}x^3 + C.$$

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