Local Extrema

Consider a function f whose domain is the set D.

- f has a local maximum at a point $c \in D$ if $f(x) \leq f(c)$ for all $x \in I$, where I is some open interval in D containing c.
- f has a local minimum at a point $c \in D$ if $f(x) \geq f(c)$ for all $x \in I$, where I is some open interval in D containing c.

a.k.a. relative extrema

Global Extrema

Consider a function f whose domain is the set D.

- f has a absolute maximum at a point $c \in D$ if $f(x) \leq f(c)$ for all $x \in D$.
- f has a absolute minimum at a point $c \in D$ if $f(x) \geq f(c)$ for all $x \in D$.

a.k.a. absolute extrema

Local Extrema

Consider a function f whose domain is the set D.

- f has a local maximum at a point $c \in D$ if $f(x) \leq f(c)$ for all $x \in I$, where I is some open interval in D containing c.
- f has a local minimum at a point $c \in D$ if $f(x) \geq f(c)$ for all $x \in I$, where I is some open interval in D containing c.

a.k.a. relative extrema

Global Extrema

Consider a function f whose domain is the set D.

- f has a absolute maximum at a point $c \in D$ if $f(x) \leq f(c)$ for all $x \in D$.
- f has a absolute minimum at a point $c \in D$ if $f(x) \geq f(c)$ for all $x \in D$.

a.k.a. absolute extrema

Local Extrema

Consider a function f whose domain is the set D.

- f has a local maximum at a point $c \in D$ if $f(x) \leq f(c)$ for all $x \in I$, where I is some open interval in D containing c.
- f has a local minimum at a point $c \in D$ if $f(x) \geq f(c)$ for all $x \in I$, where I is some open interval in D containing c.

a.k.a. relative extrema

Global Extrema

Consider a function f whose domain is the set D.

- f has a absolute maximum at a point $c \in D$ if $f(x) \leq f(c)$ for all $x \in D$.
- f has a absolute minimum at a point $c \in D$ if $f(x) \geq f(c)$ for all $x \in D$.

a.k.a. absolute extrema

Local Extrema

Consider a function f whose domain is the set D.

- f has a local maximum at a point $c \in D$ if $f(x) \leq f(c)$ for all $x \in I$, where I is some open interval in D containing c.
- f has a local minimum at a point $c \in D$ if $f(x) \geq f(c)$ for all $x \in I$, where I is some open interval in D containing c.

a.k.a. relative extrema

Global Extrema

Consider a function f whose domain is the set D.

- f has a absolute maximum at a point $c \in D$ if $f(x) \leq f(c)$ for all $x \in D$.
- f has a absolute minimum at a point $c \in D$ if $f(x) \geq f(c)$ for all $x \in D$.

a.k.a. absolute extrema

Local Extrema

Consider a function f whose domain is the set D.

- f has a local maximum at a point $c \in D$ if $f(x) \leq f(c)$ for all $x \in I$, where I is some open interval in D containing c.
- f has a local minimum at a point $c \in D$ if $f(x) \geq f(c)$ for all $x \in I$, where I is some open interval in D containing c.

a.k.a. relative extrema

Global Extrema

Consider a function f whose domain is the set D.

- f has a absolute maximum at a point $c \in D$ if $f(x) \leq f(c)$ for all $x \in D$.
- f has a absolute minimum at a point $c \in D$ if $f(x) \geq f(c)$ for all $x \in D$.

a.k.a. absolute extrema

Local Extrema

Consider a function f whose domain is the set D.

- f has a <u>local maximum</u> at a point $c \in D$ if $f(x) \leq f(c)$ for all $x \in I$, where I is some open interval in D containing c.
- f has a <u>local minimum</u> at a point $c \in D$ if $f(x) \geq f(c)$ for all $x \in I$, where I is some open interval in D containing c.

a.k.a. relative extrema

Global Extrema

Consider a function f whose domain is the set D.

- f has a <u>absolute maximum</u> at a point $c \in D$ if $f(x) \leq f(c)$ for all $x \in D$.
- f has a <u>absolute minimum</u> at a point $c \in D$ if $f(x) \geq f(c)$ for all $x \in D$.

a.k.a. absolute extrema

Local Extrema

Consider a function f whose domain is the set D.

- f has a <u>local maximum</u> at a point $c \in D$ if $f(x) \leq f(c)$ for all $x \in I$, where I is some open interval in D containing c.
- f has a <u>local minimum</u> at a point $c \in D$ if $f(x) \geq f(c)$ for all $x \in I$, where I is some open interval in D containing c.

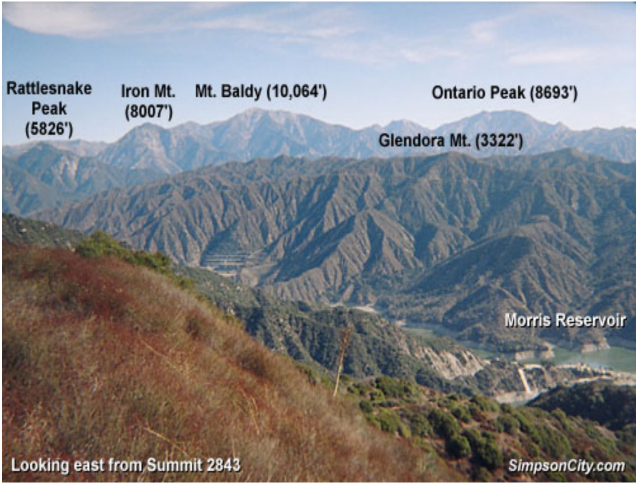
a.k.a. relative extrema

Global Extrema

Consider a function f whose domain is the set D.

- f has a <u>absolute maximum</u> at a point $c \in D$ if $f(x) \leq f(c)$ for all $x \in D$.
- f has a <u>absolute minimum</u> at a point $c \in D$ if $f(x) \geq f(c)$ for all $x \in D$.

a.k.a. absolute extrema


Rattlesnake Peak (5826')
Iron Mt. (8007')
Mt. Baldy (10,064')
Ontario Peak (8693')
Glendora Mt. (3322')
Morris Reservoir
Looking east from Summit 2843
SimpsonCity.com


Mojave Desert
San Gabriel Mtns
San Bernardino Mtns
Los
Angeles
Basin
Palm Springs
San Jacinto Mtns


MT. SAN GORGONIO
11,503

Remarks

- If f obtains either a local or global maximum at c, then $f(c)$ is the maximum value.
- If f has a global extrema at c, then it also has a local extrema at c. The converse is not true.
- There may be more than one value of c at which a global extrema is obtained.

Remarks

- If f obtains either a local or global maximum at c, then $f(c)$ is the maximum value.
- If f has a global extrema at c, then it also has a local extrema at c. The converse is not true.
- There may be more than one value of c at which a global extrema is obtained.

Remarks

- If f obtains either a local or global maximum at c, then $f(c)$ is the maximum value.
- If f has a global extrema at c, then it also has a local extrema at c. The converse is not true.
- There may be more than one value of c at which a global extrema is obtained.

Theorem 4.1.1- The Extreme Value Theorem

If f is continuous on a closed interval $[a, b]$, then f attains both a global maximum value, M, and an global minimum value, m, in $[a, b]$.

This is another existence theorem.

Theorem says: if f is continuous on $[a, b]$, there exists numbers x_M and x_m in $[a, b]$ such that $f(x_M) = M$ and $f(x_m) = m$, and $m \leq f(x) \leq M$ for all other $x \in [a, b]$.

Theorem 4.1.1- The Extreme Value Theorem

If f is continuous on a closed interval $[a, b]$, then f attains both a global maximum value, M, and an global minimum value, m, in $[a, b]$.

This is another existence theorem.

Theorem says: if f is continuous on $[a, b]$, there exists numbers x_M and x_m in $[a, b]$ such that $f(x_M) = M$ and $f(x_m) = m$, and $m \leq f(x) \leq M$ for all other $x \in [a, b]$.

Theorem 4.1.1- The Extreme Value Theorem

If f is continuous on a closed interval $[a, b]$, then f attains both a global maximum value, M, and an global minimum value, m, in $[a, b]$.

This is another existence theorem.

Theorem says: if f is continuous on $[a, b]$, there exists numbers x_M and x_m in $[a, b]$ such that $f(x_M) = M$ and $f(x_m) = m$, and $m \leq f(x) \leq M$ for all other $x \in [a, b]$.

Theorem 4.1.1- The Extreme Value Theorem

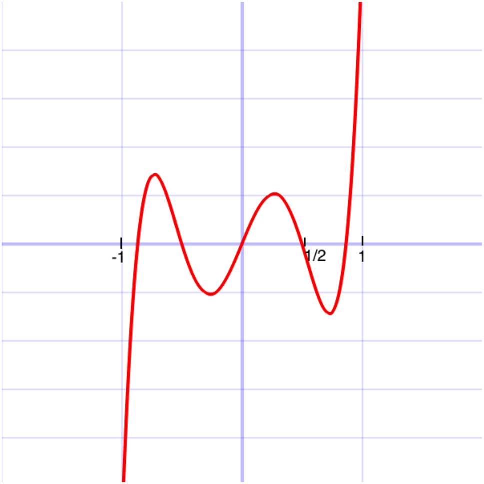
If f is continuous on a closed interval $[a, b]$, then f attains both a global maximum value, M, and an global minimum value, m, in $[a, b]$.

This is another existence theorem.

Theorem says: if f is continuous on $[a, b]$, there exists numbers x_M and x_m in $[a, b]$ such that $f(x_M) = M$ and $f(x_m) = m$, and $m \leq f(x) \leq M$ for all other $x \in [a, b]$.

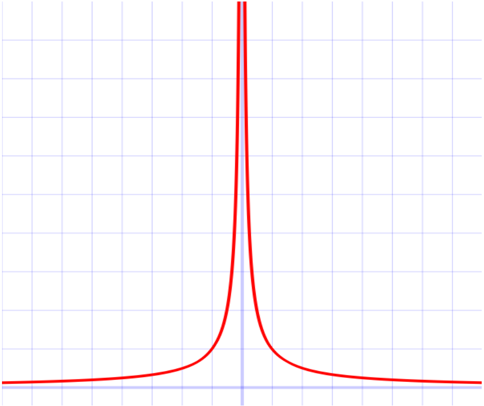
Extreme Value Theorem

Ex 1a $f(x) = 32x^5 - 32x^3 + 6x$



Extreme Value Theorem

Ex 1b $g(x) = \frac{1}{|x|}$

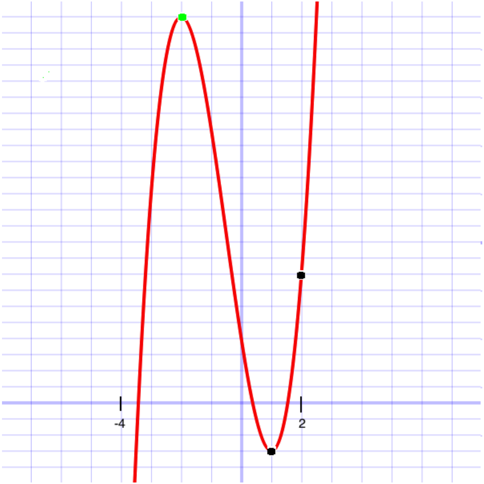


Finding Global Extrema

If f is continuous on a closed interval $[a, b]$, then to find its global extrema:

1. Find all critical points of f contained in (a, b).
2. Evaluate f at each critical point.
3. Evaluate f at the endpoints. (i.e. find $f(a)$ and $f(b)$.)
4. The largest value of $f(x)$ from steps 2 & 3 is the global maximum of f on $[a, b]$, and the smallest of the values is the global minimum.

Visualizing example 3:



Increasing/ Decreasing

Lemma 4.3.3

Suppose that f is continuous on $[a,b]$ and differentiable on (a,b). Then,

- if $f'(x) > 0$ at *each* point $x \in (a,b)$, then f is increasing on $[a,b]$.
- if $f'(x) < 0$ at *each* point $x \in (a,b)$, then f is decreasing on $[a,b]$.

Seems reasonable- we'll prove this fact next lecture.

Increasing/ Decreasing

Lemma 4.3.3

Suppose that f is continuous on $[a, b]$ and differentiable on (a, b). Then,

- if $f'(x) > 0$ at *each* point $x \in (a, b)$, then f is increasing on $[a, b]$.
- if $f'(x) < 0$ at *each* point $x \in (a, b)$, then f is decreasing on $[a, b]$.

Seems reasonable- we'll prove this fact next lecture.

Increasing/ Decreasing

Lemma 4.3.3

Suppose that f is continuous on $[a, b]$ and differentiable on (a, b). Then,

- if $f'(x) > 0$ at *each* point $x \in (a, b)$, then f is increasing on $[a, b]$.
- if $f'(x) < 0$ at *each* point $x \in (a, b)$, then f is decreasing on $[a, b]$.

Seems reasonable- we'll prove this fact next lecture.

First Derivative Test for Local Extrema

First Derivative Test

Suppose that c is a critical point of a continuous function f, and that f is differentiable at every point in some interval I containing c except possibly at c itself. Moving across the interval I from left to right.

- if $f'(x)$ changes from negative to positive at c, then f has a local minimum at c,
- if $f'(x)$ changes from positive to negative at c, then f has a local maximum at c,
- if $f'(x)$ does not change sign at c, then f has no local extremum at c.

First Derivative Test for Local Extrema

First Derivative Test

Suppose that c is a critical point of a continuous function f, and that f is differentiable at every point in some interval I containing c except possibly at c itself. Moving across the interval I from left to right.

- if $f'(x)$ changes from negative to positive at c, then f has a local minimum at c,
- if $f'(x)$ changes from positive to negative at c, then f has a local maximum at c,
- if $f'(x)$ does not change sign at c, then f has no local extremum at c.

First Derivative Test for Local Extrema

First Derivative Test

Suppose that c is a critical point of a continuous function f, and that f is differentiable at every point in some interval I containing c except possibly at c itself. Moving across the interval I from left to right.

- if $f'(x)$ changes from negative to positive at c, then f has a local minimum at c,
- if $f'(x)$ changes from positive to negative at c, then f has a local maximum at c,
- if $f'(x)$ does not change sign at c, then f has no local extremum at c.

First Derivative Test for Local Extrema

First Derivative Test

Suppose that c is a critical point of a continuous function f, and that f is differentiable at every point in some interval I containing c except possibly at c itself. Moving across the interval I from left to right.

- if $f'(x)$ changes from negative to positive at c, then f has a local minimum at c,
- if $f'(x)$ changes from positive to negative at c, then f has a local maximum at c,
- if $f'(x)$ does not change sign at c, then f has no local extremum at c.

Example 4

Consider $f(x) = 3x^4 - 8x^3 + 5$.

$$f'(x) = 12x^3 - 24x^2 = 12x^2(x-2)$$

so the critical points are $x = 0$ & $x = 2$, which are both Type I.

a) Does f have local extrema at either of these points?

b) Find the global extrema of f on $[-1, 3]$.

Example 4

Consider $f(x) = 3x^4 - 8x^3 + 5$.

$$f'(x) = 12x^3 - 24x^2 = 12x^2(x-2)$$

so the critical points are $x = 0$ & $x = 2$, which are both Type I.

a) Does f have local extrema at either of these points?

b) Find the global extrema of f on $[-1, 3]$.

Example 4

Consider $f(x) = 3x^4 - 8x^3 + 5$.

$$f'(x) = 12x^3 - 24x^2 = 12x^2(x-2)$$

so the critical points are $x = 0$ & $x = 2$, which are both Type I.

a) Does f have local extrema at either of these points?

b) Find the global extrema of f on $[-1, 3]$.

Example 4

Consider $f(x) = 3x^4 - 8x^3 + 5$.

$$f'(x) = 12x^3 - 24x^2 = 12x^2(x-2)$$

so the critical points are $x = 0$ & $x = 2$, which are both Type I.

a) Does f have local extrema at either of these points?

b) Find the global extrema of f on $[-1, 3]$.