Rolle's Theorem

Theorem 4.2.3

Suppose that $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b). If $f(a) = f(b)$, the there is at least one number $c \in (a, b)$ at which $f'(c) = 0$.

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Remarks

- This is another existence theorem.
- If the slope of of the secant line between $(a, f(a))$ and $(b, f(b))$ is zero, there must be a point $c \in (a, b)$ at which the tangent line to f is horizontal.
- If the average rate of change over $[a, b]$ is zero, then the instantaneous rate of change must be 0 at some point in (a, b).

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Mean Value Theorem

Theorem 4.2.4

Suppose $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b). There there is a least one point $c \in (a, b)$ at which

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  - then there exists at least one point in (a, b) such that the instantaneous rate of change of $f(x)$ is equal to the average rate of change of f over the interval $[a, b]$.

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Exercise

True or False?

If $f(x) = |x|$, then there is a point $c \in (-1, 2)$ such that

$$f'(c) = \frac{f(2) - f(-1)}{3}.$$

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We know for all $x < 0$, $f'(x) = -1$, and for all $x > 0, f'(x) = 1$.

Q: Why doesn't this contradict the Mean Value Theorem?

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Results following from MVT

Corollary 1

If $f'(x) = 0$ for all $x \in (a, b)$, then there is a constant k such that

$$f(x) = k, \quad \text{for all } x \in (a, b).$$

Corollary 2

If $f'(x) = g'(x)$ for all $x \in (a, b)$, then there exists a constant k such that $f(x) - g(x) = k$ for all $x \in (a, b)$.

That is $f(x) = g(x) + k$ for all $x \in (a, b)$.

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Results following from MVT

Corollary 4.3.3- Previously Presented as Lemma 4.3.3

Suppose that f is continuous on $[a, b]$ and differentiable on (a, b). Then,

- if $f'(x) > 0$ at *each* point $x \in (a, b)$, then f is increasing on $[a, b]$.
- if $f'(x) < 0$ at *each* point $x \in (a, b)$, then f is decreasing on $[a, b]$.

Results following from MVT

Corollary 4.3.3- Previously Presented as Lemma 4.3.3

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Definitions:

A function is increasing on $[a, b]$, if for any x_1 and x_2 with $a \leq x_1 < x_2 \leq b$

$$f(x_1) \leq f(x_2).$$

A function is decreasing on $[a, b]$, if for any x_1 and x_2 with $a \leq x_1 < x_2 \leq b$

$$f(x_1) \geq f(x_2).$$

Example

Let R be the region between the two concentric circles with radii r_1 and r_2, where $r_2 > r_1$. Let A be the area of the region R.

a) Draw a picture of the region and find an expression for A.

b) Why should we believe /how can we explain why the following statement is true?

> *There must be a number r such that the rectangle with width $2\pi r$ and height $r_2 - r_1$ has area exactly equal to A.*

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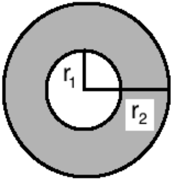
Why should we believe /how can we explain why the following statement is true?

> *There must be a number r such that the rectangle with width $2\pi r$ and height $r_2 - r_1$ has area exactly equal to A.*
> *That is, there is an r such that $2\pi r(r_2 - r_1) = \pi(r_2^2 - r_1^2)$.*



Area of a circle with radius x: $A(x) = \pi x^2$.

- A is continuous for all r.
- A is differentiable for all r.

Section Assignment #9

Work in pairs or groups of three. Turn in one sheet per group at the end of class.

1. Let f be a function which is continuous and differentiable at every real number. Show that if $f(x)$ has at least two roots (that is at least two different values of x such that $f(x) = 0$), then $f'(x)$ must have at least one root.

2. Suppose a sailboat travels 184 seamiles in a 24 hour period. Explain why at some point during this window of time the boat's speed must have exceeded 7.5 knots (seamiles per hour).