Trigonometric Functions

Trigonometric Model

On a certain day, high tide at Pacific Beach was at midnight. The water level at high tide was 9.9 feet and later at the following low tide, the tide height was 0.1 ft. The next high tide was at 12 noon.

The height of the water is given by the following function

$$h(t) = 5 + 4.9\cos((\pi/6)t),$$

where $t = 0$ corresponds to midnight.

Interpret this equation.

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Derivative of Cosine

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Will Need:

- Identity: $\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$
- Theorem 2.4.7: $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$
- Half Angle Formula: $\sin^2(\theta/2) = \frac{1-\cos(\theta)}{2}$

$$\lim_{h\to 0} \frac{\cos(x+h)-\cos(x)}{h} = \lim_{h\to 0} \frac{\cos(x)\cos(h)-\sin(x)\sin(h)-\cos(x)}{h}$$

$$= \lim_{h\to 0} \frac{\cos(x)(\cos(h)-1)-\sin(x)\sin(h)}{h}$$

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$$\begin{aligned}
\lim_{h\to 0} \frac{\cos(x+h) - \cos(x)}{h} &= \lim_{h\to 0} \frac{\cos(x)\cos(h) - \sin(x)\sin(h) - \cos(x)}{h} \\
&= \lim_{h\to 0} \frac{\cos(x)(\cos(h)-1) - \sin(x)\sin(h)}{h} \\
&= \lim_{h\to 0} \cos(x)\frac{(\cos(h)-1)}{h} - \sin(x)\frac{\sin(h)}{h}
\end{aligned}$$

By Theorem 2.4.7 $\lim_{h\to 0}\frac{\sin(h)}{h} = 1$. Therefore,

$$\lim_{h\to 0}\sin(x)\frac{\sin(h)}{h} = \sin(x).$$

By the half angle formula, $\cos(h) - 1 = -2\sin^2(h/2)$. Combined with Theorem 2.4.7 this gives

$$\lim_{h\to 0}\cos(x)\frac{\cos(h)-1}{h} = \lim_{h\to 0}\cos(x)\frac{-2\sin(h/2)}{h}\sin(h/2) = \cos(x)(0) = 0.$$

Thus,

$$\frac{d}{dx}\cos(x) = 0 - \sin(x) = -\sin(x).$$

A similar proof shows $\frac{d}{dx}\sin(x) = \cos(x)$.

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Derivatives of Trig Functions

$$\frac{d}{dx}\cos(x) = -\sin(x), \quad \text{and} \quad \frac{d}{dx}\sin(x) = \cos(x)$$

Derivatives of other trig functions?

3.5- Derivatives of Trig Functions

- $\frac{d}{dx}\cos(x) = -\sin(x)$
- $\frac{d}{dx}\sin(x) = \cos(x)$
- $\frac{d}{dx}\tan(x) = \sec^2(x) = (\sec(x))^2$
- $\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$
- $\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$
- $\frac{d}{dx}\cot(x) = -\csc^2(x)$

Gears

http://www.youtube.com/watch?v=fp9fj9cYkDs