

For  $2 \times 2$  tables, the Chi Square statistic is the square of the z-statistic used to test equality of proportions

blue = observed  
brown = expected

|                       |                       |
|-----------------------|-----------------------|
| $n_1 \hat{p}_1$       | $n_2 \hat{p}_2$       |
| $n_1 \bar{p}$         | $n_2 \bar{p}$         |
| $n_1 (1 - \hat{p}_1)$ | $n_2 (1 - \hat{p}_2)$ |
| $n_1 (1 - \bar{p})$   | $n_2 (1 - \bar{p})$   |

Hyp. Test for Proportions:

$$H_0: p_1 = p_2$$

$$H_a: p_1 \neq p_2$$

Use

$$\bar{p} = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

For SE =

$$\sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

$n_1$

$n_2$

Note  $\hat{p}_1 - \bar{p} = \left(\frac{n_2}{n_1 + n_2}\right)(\hat{p}_2 - \hat{p}_1)$

Upper  
left cell

$$\frac{(E - O)^2}{E} = \frac{\left[n_1 \left(\frac{n_2}{n_1 + n_2}\right)(\hat{p}_2 - \hat{p}_1)\right]^2}{n_1 \bar{p}}$$

$$= \left[\frac{n_1 n_2}{n_1 + n_2}\right] (\hat{p}_2 - \hat{p}_1)^2 \left[\frac{n_2}{n_1 + n_2} \cdot \frac{1}{\bar{p}}\right]$$