

linear models: $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$

b_1 is an unbiased estimator of β_1
• y_i : Treat x_i as fixed.

$$b_1 = r \frac{s_y}{s_x} = \frac{1}{(n-1)s_x^2} \sum (x_i - \bar{x})(y_i - \bar{y})$$

$$= \frac{1}{(n-1)s_x^2} \sum y_i (x_i - \bar{x}) \quad \text{since} \quad \sum (x_i - \bar{x}) = 0$$

$$E(\hat{b}_1) = \frac{1}{(n-1)s_x^2} \sum E(y_i)(x_i - \bar{x})$$

$$= \frac{1}{(n-1)s_x^2} \sum (\beta_0 + \beta_1 x_i)(x_i - \bar{x})$$

$$= \frac{\beta_1}{(n-1)s_x^2} \sum (x_i - \bar{x})(x_i - \bar{x})$$

$$= \beta_1$$