Expection of a Sum

$$
\begin{aligned}
E(X+Y) &= \Sigma_{i,j}(x_i + y_j)P(X = x_i \text{ and } Y = y_j) \\
&= \Sigma_{i,j}x_i P(X = x_i \mid Y = y_j)P(Y = y_j) \\
&\quad +\Sigma_{i,j}y_j P(Y = y_j \mid X = x_i)P(X = x_i) \\
&= \Sigma_i x_i \left(\Sigma_j P(X = x_i \mid Y = y_j)P(Y = y_j)\right) \\
&\quad +\Sigma_j y_j \left(\Sigma_i P(Y = y_j \mid X = x_i)P(X = x_i)\right) \\
&= \Sigma_i x_i P(X = x_i) \\
&\quad +\Sigma_j y_j P(Y = y_j) \\
&= E(X) + E(Y).
\end{aligned}
$$