Math 1710 Class 1

Dr. Allen Back

Aug. 28, 2009

Outline

1 Topics

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Probability (5 wks)

- Basic Ideas
- Random Variables
- Law of Large Numbers
- Normal and Binomial Distributions
- Central Limit Theorem
- Sampling Distributions

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- Correlation and Regression (2 Var)
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- Hypothesis Testing
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- (Z,t, proportions, means, two-sample)

More Refined (2 wks)

- Chi Square
- Inference for Regression

AP Stats vs. 1710

Some different emphases.

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Lecture a good guide to exam priorities.
(Some explanations will go beyond what you will be responsible for.)

Fancy Words

Random Phenomenon - Individually not predictable.
But if repeated, long term relative frequency is.

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Random Phenomenon
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Random Phenomenon -
e.g. rolling a die

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e.g. $S = \{1, 2, 3, 4, 5, 6\}$.

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Event E - a subset of the sample space – a set of outcomes.

e.g. $E = \{2, 4, 6\}$

"Rolling an even number."

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"Rolling a total of 1 head."

Probability Assignment on a Sample Space S

We won't usually be this formal!

(1) $0 \leq$ **P(Outcome)** ≤ 1**.**

(2) Σ P(All Outcomes in S)$= 1$.

(3) Σ P(Outcomes in an Event)=P(Event).

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Probability Properties

(1) $0 \leq P(E) \leq 1$ **for any event** E**.**
(2) $P(\phi) = 0$, $P(S) = 1$.
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A Probability Assignment

English language frequencies:

E 13.0%	H 3.5%	W 1.6
T 9.3%	L 3.5%	V 1.3%
N 7.8%	C 3.0%	B 0.9%
R 7.7%	F 2.8%	X 0.5%
O 7.4%	P 2.7%	K 0.3%
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An Event E: e.g. being a vowel. Formally: E={A,E,I,O,U}

$P(E) = 13.0 + 7.4 + 7.4 + 7.3 + 2.7\% = 37.8\%.$

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$E_2 = \{H, E, L, O\}.$

$P(E_2) = 3.5 + 13 + 3.5 + 7.4\% = 37.8\%.$

Addition Rule

$E_2 = \{H, E, L.O\}$.
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$E_3 = \{B, A, D\}$.
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Union ($\cup$) corresponds to "or'.'

Equally Likely Outcomes

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If so, each has probability $\frac{1}{n}$.
So the sample space $S = \{HH, HT, TH, TT\}$ for tossing 2 coins shows us

Num Heads	**Prob.**
0	$\frac{1}{4}$
1	$\frac{1}{2}$
2	$\frac{1}{4}$

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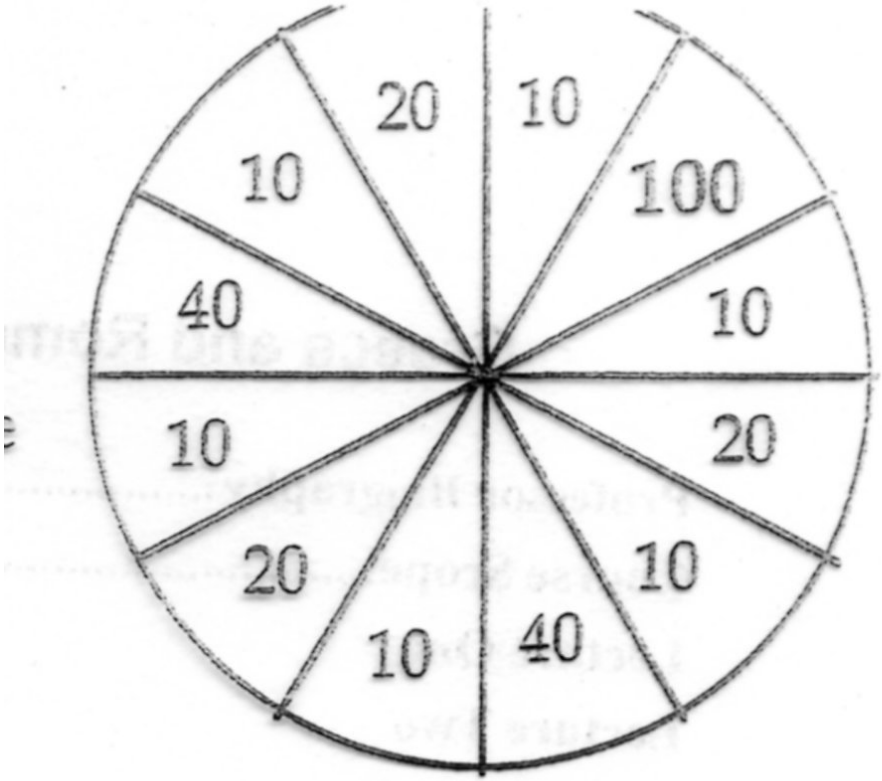
Intersection ($\cap$) corresponds to "and'.

Spinner Prob Asst

Discount	Probability
10%	$6/12 = .5$
20%	$3/12 = .25$
40%	$2/12 = .17$
100%	$1/12 = .08$

Customers Spin a Wheel

12 equal size areas.
Wheel gives customer discount in %.



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(1) Prob of at least a 40% discount?

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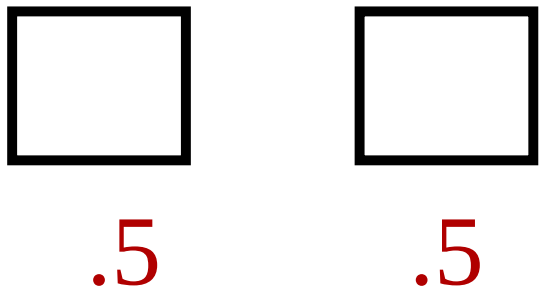

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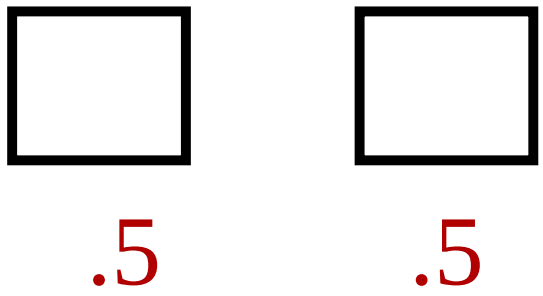

$.5^2 = .25.$

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(3) Prob 3 consecutive customers get 20%?

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P(all 20% or less)$=\left(\frac{3}{4}\right)^4$

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(5) Prob that first 100% is fifth customer?

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$\left(\frac{11}{12}\right)^4 \left(\frac{1}{12}\right)$

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(6) Chance that at least one of the first six shoppers gets less than a 100% discount?

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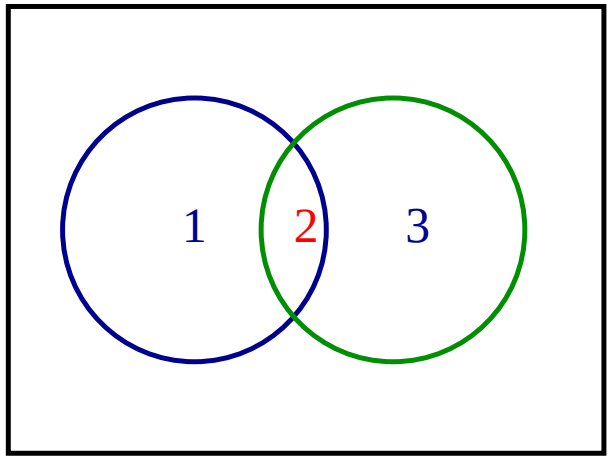
P(0 get less than 100%)=P(all 100%)=$\left(\frac{11}{12}\right)^6$.

So complement rule says answer to (6) is $1-\left(\frac{11}{12}\right)^6$.

Venn Diagram Picture



$$A = \{1, 2\} \quad B = \{2, 3\}$$

$$A \cap B = \{2\} \quad A \cup B = \{1, 2, 3\}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$1, 2, 3 \qquad 1, 2 \qquad 2, 3 \qquad 2$$

Playing Sports

Suppose

- 30% play soccer
- 20% basketball
- 12% both.

Probability of playing at least one?

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Soln: $.30 + .20 - .12 = .38$.

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