Math 1710 Class 2

Prob. incl Conditional

Aug. 31, 2009

Outline

1 Last Time

Outline

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Key Properties

(1) Σ P(Outcomes in an Event)=P(Event).

(2) $P(A \cup B) = P(A) + P(B)$ if A and B are disjoint.

(3) $P(A^c) = 1 - P(A)$

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So the sample space $S = \{HH, HT, TH, TT\}$ for tossing 2 coins shows us

Num Heads	**Prob.**
0	$\frac{1}{4}$
1	$\frac{1}{2}$
2	$\frac{1}{4}$

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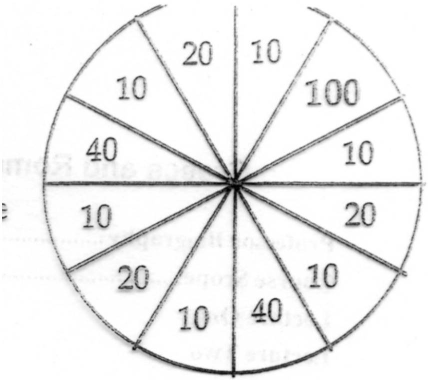
Intersection ($\cap$) corresponds to "and".

Customers Spin a Wheel

12 equal size areas.
Wheel gives customer discount in %.



Spinner Prob Asst

Discount	Probability
10%	6/12=.50
20%	3/12=.25
40%	2/12=.17
100%	1/12=.08

(1) Prob of at least a 40% discount?

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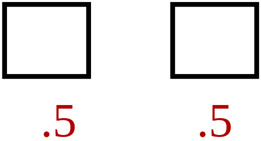

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$\square \quad \square$

$.5 \quad .5$

$.5^2 = .25.$

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(3) Prob 3 consecutive customers get 20%?

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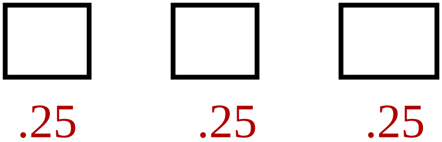

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(4) Prob none of first four get over 20%?

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(4) Prob none of first four get over 20%?

P(all 20% or less)$=\left(\frac{3}{4}\right)^4$

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(5) Prob that first 100% is fifth customer?

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100%	1/12=.08

(5) Prob that first 100% is fifth customer?

11/12	11/12	11/12	11/12	1/12

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(5) Prob that first 100% is fifth customer?

☐ ☐ ☐ ☐ ☐

11/12 11/12 11/12 11/12 1/12

$\left(\frac{11}{12}\right)^4 \left(\frac{1}{12}\right)$

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Discount	Probability
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(6) Chance that at least one of the first six shoppers gets less than a 100% discount?

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Hard b/c could be 1 or 2 or . . .

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P(0 get less than 100%)=P(all 100%)=$\left(\frac{1}{12}\right)^6$.

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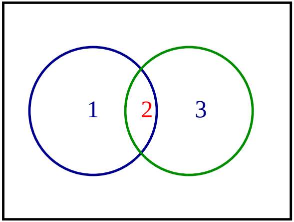
P(0 get less than 100%)=P(all 100%)=$\left(\frac{1}{12}\right)^6$.

So complement rule $\Longrightarrow$ answer to (6) is $1-\left(\frac{1}{12}\right)^6$.

Venn Diagram Picture



$$A = \{1, 2\} \qquad B = \{2, 3\}$$

$$A \cap B = \{2\} \qquad A \cup B = \{1, 2, 3\}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$1, 2, 3 \qquad 1, 2 \qquad 2, 3 \qquad 2$$

Playing Sports

Suppose

- 30% play soccer
- 20% basketball
- 12% both.

Probability of playing at least one?

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Soln: $.30 + .20 - .12 = .38.$

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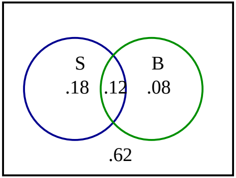

Ace or Red

One card is drawn.
Probability of an ace or red card?

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Direct counting possible:
26 red cards + 2 black aces out of 52.

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Inclusion/Exclusion:
P(Ace or Red) = P(Ace) +P(Red) - P(Ace and Red)

$$= \frac{4}{52} + \frac{26}{52} - \frac{2}{52} = \frac{28}{52}$$

Smallest Card a Queen

Two cards dealt.
Prob(smallest a queen)?
(Q viewed as smallest when two Q's.
Ace viewed as high.)

Smallest Card a Queen

Two cards dealt.
Prob(smallest a queen)?

Approach:

A = event 1st card =Q & 2nd a Q,K or A.

B = event 2nd card =Q & 1st a Q,K or A.

P(A or B) wanted!

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Two cards dealt.
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Approach:

A = event 1st card =Q & 2nd a Q,K or A.

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P(A or B)=P(A)+P(B)-P(A and B)

Smallest Card a Queen

Two cards dealt.
Prob(smallest a queen)?

A = event 1st card =Q & 2nd a Q,K or A.

$$P(A) = \frac{4}{52} \cdot \frac{11}{51} = \frac{44}{51 \cdot 52}$$

Smallest Card a Queen

Two cards dealt.
Prob(smallest a queen)?

B = event 2nd card =Q & 1st a Q,K or A.

event B

>=Q	Q
11/51	4/52

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$$P(B) = \frac{44}{51 \cdot 52} \text{ too.}$$

Smallest Card a Queen

Two cards dealt.
Prob(smallest a queen)?

$$P(\text{A and B}) = \frac{4}{52} \cdot \frac{3}{51} = \frac{12}{51 \cdot 52}$$

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P(A or B)=P(A)+P(B)-P(A and B)

$$\text{So } P(A \text{ or } B) = \frac{44}{51 \cdot 52} + \frac{44}{51 \cdot 52} - \frac{12}{51 \cdot 52} = \frac{76}{51 \cdot 52}$$

Definition

"Probability of A given B."

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

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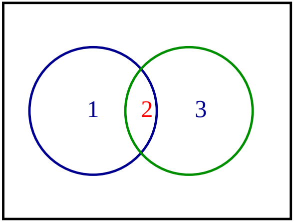

$$A = \{1, 2\} \qquad B = \{2, 3\}$$

$$A \cap B = \{2\}$$

Life Tables

Appendix C

Life Table

Number of survivors at single years of Age, out of 100,000 Born Alive, by Race and Sex: United States, 1990.

	All races					All races		
Age	Both sexes	Male	Female		Age	Both sexes	Male	Female
0	100000	100000	100000		43	94707	92840	96626
1	99073	98969	99183		44	94453	92505	96455
2	99008	98894	99128		45	94179	92147	96268

Really a table of probabilities;
e.g. P(woman lives to 45)=.963.
Will write as P(45).

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59	86596	82678	90571	79	50026	39872	59796
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61	84590	80194	89033	81	44232	33811	54186

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$$P(80|60) = \frac{P(80 \cap 60)}{P(60)}$$

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$$P(60|80) = \frac{P(80 \cap 60)}{P(80)}$$

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$P(60|80) \neq P(80|60)$.

Independence

A and B are independent events if

$$P(A|B) = P(A)$$

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10% have oil leaks
1% both.
Are sticky valves and oil leaks independent?

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$$P(sticky|leaky) = \frac{P(both)}{P(leaky)} = \frac{.01}{.10} = .10 = P(sticky).$$

So they are independent.

Independence

A and B are independent events if

$$P(A|B) = P(A)$$

Box has 2 white tickets labeled 1
2 black labeled 1
1 white labeled 8
1 black one labeled 6.
Are drawing a 1 and color black independent?

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$$P(1|black) = \frac{2}{3} \qquad P(1) = \frac{4}{6}$$

Equal, so they are independent.

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Box has 2 white tickets labeled 1
2 black labeled 1
1 white labeled 8
1 black one labeled 6.
Are drawing an 8 and color black independent?

$$P(8|black) = \frac{0}{3} \qquad P(8) = \frac{1}{6}$$

Not equal, so not independent.