

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Math 1710 Class 33

t - Inference incl. 2-Sample and MP
Dr. Back

Nov. 13, 2009

Using Table T

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

One tail probability		0.10	0.05	0.025	0.01	0.005
Table T	df					
Values of t_{α}	1	3.078	6.314	12.706	31.821	63.657
	2	1.886	2.920	4.303	6.965	9.925
	3	1.638	2.353	3.182	4.541	5.841
	4	1.533	2.132	2.776	3.747	4.604

	17	1.333	1.740	2.110	2.567	2.898
	18	1.330	1.734	2.101	2.552	2.878
	19	1.328	1.729	2.093	2.539	2.861
	20	1.325	1.725	2.086	2.528	2.845
	21	1.323	1.721	2.080	2.518	2.831

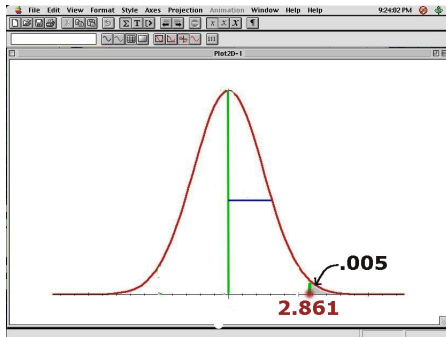
These are all critical values t^* .

For example $P(t > 1.328) = .10$ for the t distribution with 19 df.

Using Table T

Our t-statistic of -3.49 is more extreme than any on the $df=19$ row of the table.

The picture



shows what the critical value $t^* = 2.861$ for a tail prob. of $.005$ means.

Using Table T

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

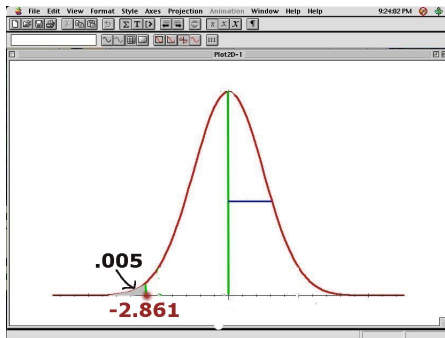
Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

So by symmetry



$P(T < -2.861) = .005$ as well.

Using Table T

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

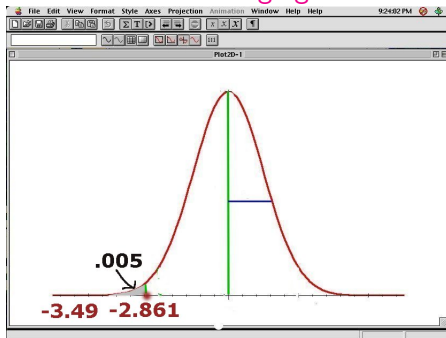
Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

So our tail probability and p-value are both less than $.005$ and we reject the null. The machine does appear to be shortchanging.



Using Table T

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Sps. our t-statistic had been 2.00 with the same 1-sided hypotheses

- $H_0: \mu = 10$ (or $\mu \geq 10$)
- $H_a: \mu < 10$

What P-value would we report?

Using Table T

Sps. our t-statistic had been 2.00 with the same 1-sided hypotheses

- $H_0: \mu = 10$ (or $\mu \geq 10$)
- $H_a: \mu < 10$

What P-value would we report?

One tail probability		0.10	0.05	0.025	0.01	0.005
df	Table T					
	Values of t_α					
1		3.078	6.314	12.706	31.821	63.657
2		1.886	2.920	4.303	6.965	9.925
3		1.638	2.353	3.182	4.541	5.841
4		1.533	2.132	2.776	3.747	4.604
...	
17		1.333	1.740	2.110	2.567	2.898
18		1.330	1.734	2.101	2.557	2.878
19		1.328	1.729	2.093	2.539	2.861
20		1.325	1.725	2.086	2.528	2.845
21		1.323	1.721	2.080	2.518	2.831
...	

Answer: A tail probability and P-value of between .025 and .05.

Using Table T

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Sps. instead our t-statistic had been 2.00 with 2-sided hypotheses

- $H_0: \mu = 10$
- $H_a: \mu \neq 10$

What P-value would we report?

Using Table T

Sps. instead our t-statistic had been 2.00 with 2-sided hypotheses

- $H_0: \mu = 10$
- $H_a: \mu \neq 10$

What P-value would we report?

One tail probability		0.10	0.05	0.025	0.01	0.005
Table T	df					
Values of t_α	1	3.078	6.314	12.706	31.821	63.657
	2	1.886	2.920	4.303	6.965	9.925
	3	1.638	2.353	3.182	4.541	5.841
	4	1.533	2.132	2.776	3.747	4.604
	17	1.333	1.740	2.110	2.567	2.898
	18	1.330	1.734	2.101	2.557	2.878
	19	1.328	1.729	2.093	2.539	2.861
	20	1.325	1.725	2.086	2.528	2.845
	21	1.323	1.721	2.080	2.518	2.831

Answer: Our tail probability is still between .025 and .05 but our P-value is now between .05 and .10.

Using Table T

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Sps. instead our t-statistic had been 2.00 with 2-sided hypotheses

- $H_0: \mu = 10$
- $H_a: \mu \neq 10$

What P-value would we report?

In using the table, always be conservative. If **you want $df=138$ and there is a $df=120$ as well as a $df=140$ row**, the principle of being conservative means to **use the $df=120$ row**. When you say “I am 95% confident . . .” or “I reject H_0 ”, you’ll be delivering what you promised; with better tables you’d just be able to report your level of confidence as somewhat higher.

By Calculator

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

TI 83/84: 2nd \rightarrow distr \rightarrow tcdf(-100,-2,19) to find
 $P(T < -2)$ for a t-distribution with $df = 19$.

By Calculator

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

TI 83/84: 2nd \rightarrow distr \rightarrow tcdf(-100,-2,19) to find $P(T < -2)$ for a t-distribution with $df = 19$.

TI 89: catalog \rightarrow F3 \rightarrow 2nd-alpha t ... tcdf(-100,-2,19) to find $P(T < -2)$ for a t-distribution with $df = 19$.

A Sample Size Problem

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Suppose we used a pilot sample of size 25 and came up with $\bar{x} = 815$ and $s = 1$ assuming that all the conditions for t-inference were met. **How large a sample size should we choose for a larger sample to achieve a 95% CI with a margin of error of approximately .1?**

A Sample Size Problem

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Suppose we used a pilot sample of size 25 and came up with $\bar{x} = 815$ and $s = 1$ assuming that all the conditions for t-inference were met. **How large a sample size should we choose for a larger sample to achieve a 95% CI with a margin of error of approximately .1?**

Remember the margin of error MOE is half the width of the CI.
So

$$MOE = t^* \frac{s}{\sqrt{n}}$$

There are two complications in using the formula:

- t^* depends non-linearly on n
- s varies from sample to sample

A Sample Size Problem

Suppose we used a pilot sample of size 25 and came up with $\bar{x} = 815$ and $s = 1$ assuming that all the conditions for t-inference were met. **How large a sample size should we choose for a larger sample to achieve a 95% CI with a margin of error of approximately .1?**

So

$$MOE = t^* \frac{s}{\sqrt{n}}.$$

There are two complications in using the formula:

- t^* depends non-linearly on n
- s varies from sample to sample

The second factor could be kept track of using the Chi Square distribution which we will use (for other purposes) in chapter 26.

The textbook treatment is to just make believe s will likely not change much in the next sample.

A Sample Size Problem

Suppose we used a pilot sample of size 25 and came up with $\bar{x} = 815$ and $s = 1$ assuming that all the conditions for t-inference were met. **How large a sample size should we choose for a larger sample to achieve a 95% CI with a margin of error of approximately .1?**

So

$$MOE = t^* \frac{s}{\sqrt{n}}$$

There are two complications in using the formula:

- t^* depends non-linearly on n
- s varies from sample to sample

Assuming s does not change,

$$n = \left(\frac{t^* s}{MOE} \right)^2$$

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

A Sample Size Problem

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Suppose we used a pilot sample of size 25 and came up with $\bar{x} = 815$ and $s = 1$ assuming that all the conditions for t-inference were met. **How large a sample size should we choose for a larger sample to achieve a 95% CI with a margin of error of approximately .1?**

Assuming s does not change,

$$n = \left(\frac{t^* s}{MOE} \right)^2$$

To deal with t^* , two possibilities:

- Use z^* as a rough estimate of t^* . (too low)
- Or use the t^* from our sample of size 25 as a too high estimate.

A Sample Size Problem

Suppose we used a pilot sample of size 25 and came up with $\bar{x} = 815$ and $s = 1$ assuming that all the conditions for t-inference were met. **How large a sample size should we choose for a larger sample to achieve a 95% CI with a margin of error of approximately .1?**

Assuming s does not change,

$$n = \left(\frac{t^* s}{MOE} \right)^2$$

The first method using $z^* = 1.96$ gives

$$n = \left(\frac{1.96 \cdot 1}{.1} \right)^2 = 19.6^2 = 384.16.$$

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

A Sample Size Problem

Suppose we used a pilot sample of size 25 and came up with $\bar{x} = 815$ and $s = 1$ assuming that all the conditions for t-inference were met. **How large a sample size should we choose for a larger sample to achieve a 95% CI with a margin of error of approximately .1?**

Assuming s does not change,

$$n = \left(\frac{t^* s}{MOE} \right)^2$$

For the $n = 25$ of our pilot study, $df = 24$ and $t^* = 2.064$. So the second method gives

$$n = \left(\frac{2.064 \cdot 1}{.1} \right)^2 = 20.64^2 = 426.$$

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

A Sample Size Problem

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Suppose we used a pilot sample of size 25 and came up with $\bar{x} = 815$ and $s = 1$ assuming that all the conditions for t-inference were met. **How large a sample size should we choose for a larger sample to achieve a 95% CI with a margin of error of approximately .1?**

Either of these would be a reasonable approximate answer. Your textbook points out we could use either of the above as a rough estimate of n , determine t^* , and get a better approx. of n . **This effect is much smaller than the variation in s from sample to sample and should be viewed as entirely optional.**

A Sample Size Problem

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Suppose we used a pilot sample of size 25 and came up with $\bar{x} = 815$ and $s = 1$ assuming that all the conditions for t-inference were met. **How large a sample size should we choose for a larger sample to achieve a 95% CI with a margin of error of approximately .1?**

Assuming s does not change,

$$n = \left(\frac{t^* s}{MOE} \right)^2$$

$n = 384$ means use the $df = 250$ line and $t^* = 1.969$.

$n = 426$ means use the $df = 400$ line and $t^* = 1.966$.

Being conservative . . .

A Sample Size Problem

Suppose we used a pilot sample of size 25 and came up with $\bar{x} = 815$ and $s = 1$ assuming that all the conditions for t-inference were met. **How large a sample size should we choose for a larger sample to achieve a 95% CI with a margin of error of approximately .1?**

Assuming s does not change,

$$n = \left(\frac{t^*s}{MOE} \right)^2$$

Using $t^* = 1.969$,

$$n = \left(\frac{1.969 \cdot 1s}{.1} \right)^2 = 19.69^2 = 387.7.$$

($19.66^2 = 386.52$.)

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Haiku Poetry: Intrinsic or Extrinsic Motivation Better?

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

haiku: A Japanese poem composed of three unrhymed lines of five, seven, and five syllables. Haiku often reflect on some aspect of nature.

Haiku Poetry: Intrinsic or Extrinsic Motivation Better?

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

How can you study this?

Haiku Poetry: Intrinsic or Extrinsic Motivation Better?

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

How can you study this?

Randomized Experiment on Creativity in poetry:

Grades on creativity recorded. (Higher is better.)

Both groups filled out questionnaires before writing poetry.

- Intrinsic group first did questionnaire emphasizing intrinsic motivations.
- Extrinsic group first did questionnaire emphasizing extrinsic motivations.

Haiku Poetry: Intrinsic or Extrinsic Motivation Better?

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

How can you study this?

People then wrote Haiku poems which were graded by experienced poets.

Haiku Poetry: Intrinsic or Extrinsic Motivation Better?

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

How can you study this?

The Results

Group	n	\bar{x}	s
Intrinsic	24	19.88	4.44
Extrinsic	23	15.73	5.25

Haiku Poetry: Intrinsic or Extrinsic Motivation Better?

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

How can you study this?

Notation: Let μ_i and μ_e denote the respective mean haiku scores for the intrinsically and the extrinsically motivated groups.

Haiku Poetry: Intrinsic or Extrinsic Motivation Better?

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

How can you study this?

Notation: Let μ_i and μ_e denote the respective mean haiku scores for the intrinsically and the extrinsically motivated groups.

Hypotheses:

- $H_0: \mu_i = \mu_e$
- $H_a: \mu_i \neq \mu_e$

Haiku Poetry: Intrinsic or Extrinsic Motivation Better?

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

How can you study this?

Hypotheses:

- $H_0: \mu_i = \mu_e$
- $H_a: \mu_i \neq \mu_e$

Under H_0 , the sampling distribution of $\bar{x}_i - \bar{x}_e$ has mean 0 and approx. std deviation

$$SE(\bar{x}_i - \bar{x}_e) = \sqrt{\frac{s_i^2}{n_i} + \frac{s_e^2}{n_e}}$$

Haiku Poetry: Intrinsic or Extrinsic Motivation Better?

How can you study this?

Hypotheses:

- $H_0: \mu_i = \mu_e$
- $H_a: \mu_i \neq \mu_e$

Under H_0 , the sampling distribution of $\bar{x}_i - \bar{x}_e$ has mean 0 and approx. std deviation

$$SE(\bar{x}_i - \bar{x}_e) = \sqrt{\frac{s_i^2}{n_i} + \frac{s_e^2}{n_e}}$$

$$SE(\bar{x}_i - \bar{x}_e) = \sqrt{\frac{4.44^2}{24} + \frac{5.25^2}{23}} = 1.42.$$

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Haiku Poetry: Intrinsic or Extrinsic Motivation Better?

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

How can you study this?

Hypotheses:

- $H_0: \mu_i = \mu_e$
- $H_a: \mu_i \neq \mu_e$

$$SE(\bar{x}_i - \bar{x}_e) = \sqrt{\frac{4.44^2}{24} + \frac{5.25^2}{23}} = 1.42.$$

Our t-statistic is

$$t = \frac{19.88 - 15.73}{1.42} = 2.92.$$

Haiku Poetry: Intrinsic or Extrinsic Motivation Better?

How can you study this?

Hypotheses:

- $H_0: \mu_i = \mu_e$
- $H_a: \mu_i \neq \mu_e$

$$SE(\bar{x}_i - \bar{x}_e) = \sqrt{\frac{4.44^2}{24} + \frac{5.25^2}{23}} = 1.42.$$

Our t-statistic is

$$t = \frac{19.88 - 15.73}{1.42} = 2.92.$$

For homework and hand computation on exams, we suggest you use df the smaller of 24-1 and 23-1 to interpret the t-statistic. *To be explained later.*

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Haiku Poetry: Intrinsic or Extrinsic Motivation Better?

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

How can you study this?

Hypotheses:

- $H_0: \mu_i = \mu_e$

- $H_a: \mu_i \neq \mu_e$

With $df = 22$, 2.92 is more extreme than the biggest t^* in table T (which is 2.819) so our tail probability is less than .005 and our P-value is less than .01.

Haiku Poetry: Intrinsic or Extrinsic Motivation Better?

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

How can you study this?

Hypotheses:

- $H_0: \mu_i = \mu_e$
- $H_a: \mu_i \neq \mu_e$

With $df = 22$, 2.92 is more extreme than the biggest t^* in table T (which is 2.819) so our tail probability is less than .005 and our P-value is less than .01.

We reject H_0 . Intrinsic vs. extrinsic motivation does seem to make a difference.

Haiku Poetry: Intrinsic or Extrinsic Motivation Better?

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

How can you study this?

Hypotheses:

- $H_0: \mu_i = \mu_e$
- $H_a: \mu_i \neq \mu_e$

We reject H_0 . Intrinsic vs. extrinsic motivation does seem to make a difference.

A 95% CI for $\mu_i \neq \mu_e$ would be

$$19.88 - 15.73 \pm 2.074 \cdot 1.42 = 4.15 \pm 2.95 = (1.20, 7.10).$$

Haiku Poetry: Intrinsic or Extrinsic Motivation Better?

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

How can you study this?

Hypotheses:

- $H_0: \mu_i = \mu_e$
- $H_a: \mu_i \neq \mu_e$

Scope of the Inference:

- Causal relationship seems to be established.
- Doesn't say much about a population since not an SRS.

Satterthwaite

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

In the 2-independent sample case, even if the individual data is normal, the difference $\bar{x} - \bar{y}$ does *not* follow a t-distribution unless $\sigma_x = \sigma_y$.

Satterthwaite

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

In the 2-independent sample case, even if the individual data is normal, the difference $\bar{x} - \bar{y}$ does *not* follow a t-distribution unless $\sigma_x = \sigma_y$.

But the sampling distribution of $\bar{x} - \bar{y}$ can be well approximated by a t-distribution with the following degrees of freedom:

Satterthwaite

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

In the 2-independent sample case, even if the individual data is normal, the difference $\bar{x} - \bar{y}$ does *not* follow a t-distribution unless $\sigma_x = \sigma_y$.

$$df = \frac{\left(\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}\right)^2}{\frac{1}{n_x-1} \left(\frac{s_x^2}{n_x}\right)^2 + \frac{1}{n_y-1} \left(\frac{s_y^2}{n_y}\right)^2}$$

Satterthwaite

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

In the 2-independent sample case, even if the individual data is normal, the difference $\bar{x} - \bar{y}$ does *not* follow a t-distribution unless $\sigma_x = \sigma_y$.

$$df = \frac{\left(\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}\right)^2}{\frac{1}{n_x-1} \left(\frac{s_x^2}{n_x}\right)^2 + \frac{1}{n_y-1} \left(\frac{s_y^2}{n_y}\right)^2}$$

Let $\min(n_x - 1, n_y - 1)$ denote the smallest of $n_x - 1$ and $n_y - 1$. This df is always between $\min(n_x - 1, n_y - 1)$ and $n_x + n_y - 2$.

Satterthwaite

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

In the 2-independent sample case, even if the individual data is normal, the difference $\bar{x} - \bar{y}$ does *not* follow a t-distribution unless $\sigma_x = \sigma_y$.

$$df = \frac{\left(\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}\right)^2}{\frac{1}{n_x-1} \left(\frac{s_x^2}{n_x}\right)^2 + \frac{1}{n_y-1} \left(\frac{s_y^2}{n_y}\right)^2}$$

Special Case $s_x = s_y = s$, $n_x = n_y = n$.

$$df = \frac{\left(\frac{2s^2}{n}\right)^2}{\frac{1}{n-1} \frac{2s^4}{n^2}} = 2(n-1).$$

Satterthwaite

In the 2-independent sample case, even if the individual data is normal, the difference $\bar{x} - \bar{y}$ does *not* follow a t-distribution unless $\sigma_x = \sigma_y$.

$$df = \frac{\left(\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}\right)^2}{\frac{1}{n_x-1} \left(\frac{s_x^2}{n_x}\right)^2 + \frac{1}{n_y-1} \left(\frac{s_y^2}{n_y}\right)^2}$$

Special Case $\frac{s_x^2}{n_x} \ll \frac{s_y^2}{n_y}$

$$df = \frac{\left(\frac{s_y^2}{n_y}\right)^2}{\frac{1}{n_y-1} \left(\frac{s_y^2}{n_y}\right)^2} = n_y - 1.$$

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Satterthwaite

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

In the 2-independent sample case, even if the individual data is normal, the difference $\bar{x} - \bar{y}$ does *not* follow a t-distribution unless $\sigma_x = \sigma_y$.

$$df = \frac{\left(\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}\right)^2}{\frac{1}{n_x-1} \left(\frac{s_x^2}{n_x}\right)^2 + \frac{1}{n_y-1} \left(\frac{s_y^2}{n_y}\right)^2}$$

Calculators and computers will use this df formula. *For hand computation on HW and exams, we suggest you never use this formula. Instead use the conservative $\min(n_x - 1, n_y - 1)$.*

Satterthwaite

In the 2-independent sample case, even if the individual data is normal, the difference $\bar{x} - \bar{y}$ does *not* follow a t-distribution unless $\sigma_x = \sigma_y$.

$$df = \frac{\left(\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}\right)^2}{\frac{1}{n_x-1} \left(\frac{s_x^2}{n_x}\right)^2 + \frac{1}{n_y-1} \left(\frac{s_y^2}{n_y}\right)^2}$$

Calculators and computers will use this df formula. *For hand computation on HW and exams, we suggest you never use this formula. Instead use the conservative $\min(n_x - 1, n_y - 1)$.* Your t^* will be a little too big so your CI will be a little too wide and your HT a little harder to show statistical significance.

But this is not so bad.

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Satterthwaite

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

In the 2-independent sample case, even if the individual data is normal, the difference $\bar{x} - \bar{y}$ does *not* follow a t-distribution unless $\sigma_x = \sigma_y$.

$$df = \frac{\left(\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}\right)^2}{\frac{1}{n_x-1} \left(\frac{s_x^2}{n_x}\right)^2 + \frac{1}{n_y-1} \left(\frac{s_y^2}{n_y}\right)^2}$$

In your published papers and lab reports, use a computer which will likely automatically use this formula.

Satterthwaite

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

In the 2-independent sample case, even if the individual data is normal, the difference $\bar{x} - \bar{y}$ does *not* follow a t-distribution unless $\sigma_x = \sigma_y$.

$$df = \frac{\left(\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}\right)^2}{\frac{1}{n_x-1} \left(\frac{s_x^2}{n_x}\right)^2 + \frac{1}{n_y-1} \left(\frac{s_y^2}{n_y}\right)^2}$$

Satterthwaite derived this formula using the fact that the exact mean and variance of the sampling distribution of $\bar{x} - \bar{y}$ is easy to calculate. He asked, “What df in the t-distribution would give the right μ and σ .” The answer is the above.

Satterthwaite

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

In the 2-independent sample case, even if the individual data is normal, the difference $\bar{x} - \bar{y}$ does *not* follow a t-distribution unless $\sigma_x = \sigma_y$.

$$df = \frac{\left(\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}\right)^2}{\frac{1}{n_x-1} \left(\frac{s_x^2}{n_x}\right)^2 + \frac{1}{n_y-1} \left(\frac{s_y^2}{n_y}\right)^2}$$

$$f(x) = \frac{\Gamma\left(\frac{d+1}{2}\right)}{\Gamma\left(\frac{d}{2}\right)} \frac{1}{\sqrt{d\pi}} \frac{1}{\left(1 + \frac{x^2}{d}\right)^{\frac{d+1}{2}}}$$

Data Desk View of Creativity

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Summary of **INTRINSIC:SCORE**
No Selector

Percentile 25

Count	24
Mean	19.8833
Median	20.4
StdDev	4.43951
Range	17.7
IntQRange	5.05
Lower ith %tile	17.35
Upper ith %tile	22.4

Data Desk View of Creativity

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Summary of **EXTRINSIC:SCORE**
No Selector

Percentile 25

Count	23
Mean	15.7391
Median	17.2
StdDev	5.2526
Range	19
IntQRange	7
Lower ith %tile	12.075
Upper ith %tile	19.075

Data Desk View of Creativity

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

**Data Desk
View of
Creativity**

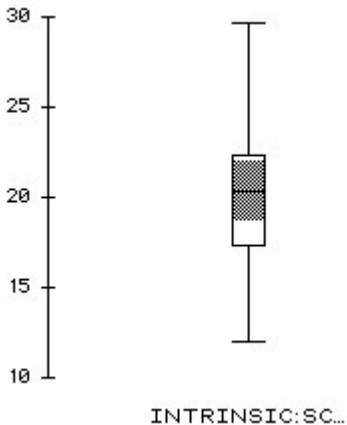
Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness



Data Desk View of Creativity

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

**Data Desk
View of
Creativity**

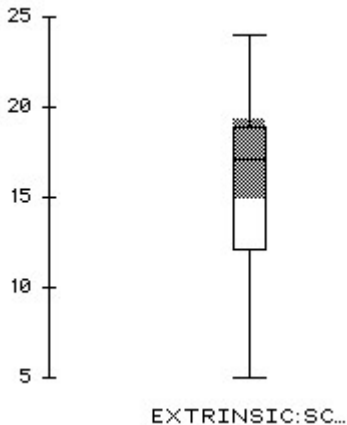
Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness



Data Desk View of Creativity

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

**Data Desk
View of
Creativity**

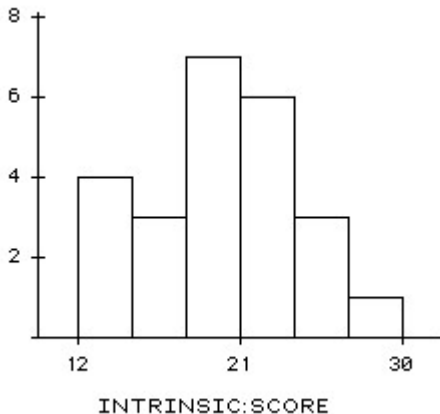
Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness



Data Desk View of Creativity

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

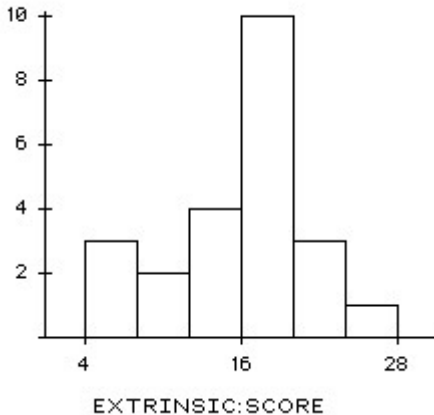
Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness



Data Desk View of Creativity

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

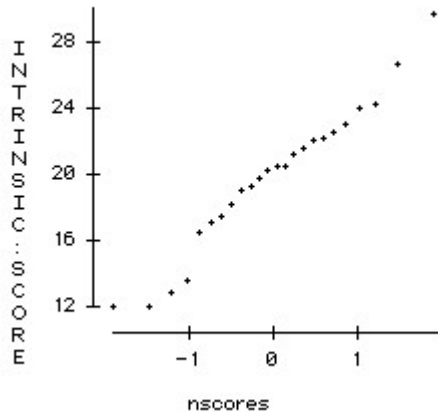
Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness



Data Desk View of Creativity

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

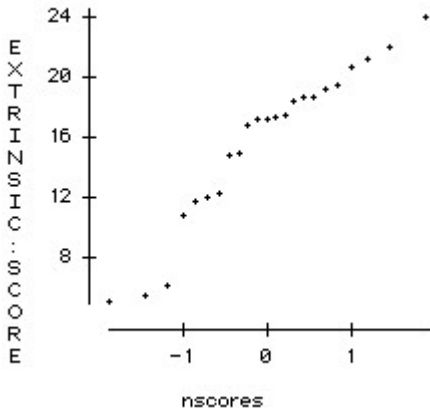
Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness



Data Desk View of Creativity

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

t-Interval for Individual μ 's

No Selector

Individual Confidence 95.00%

Bounds: Lower Bound $< \mu <$ Upper Bound

With 95.00% Confidence, 18.008691 $< \mu$ (INTRINSIC:SCORE) $<$ 21.757975

Data Desk View of Creativity

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

t-Interval for Individual μ 's

No Selector

Individual Confidence 95.00%

Bounds: Lower Bound $< \mu <$ Upper Bound

With 95.00% Confidence, 13.467738 $< \mu <$ (EXTRINSIC:SCORE) $<$ 18.010523

Data Desk View of Creativity

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

2-Sample t-Interval for $\mu_1 - \mu_2$

No Selector

Individual Confidence 95.00%

Bounds: Lower Bound $< \mu_1 - \mu_2 <$ Upper Bound

With 95.00% Confidence, $-7.0108029 < \mu(\text{EXTRINSIC:SCORE}) - \mu(\text{INTRINSIC:SCORE}) < -1.2776029$

Data Desk View of Creativity

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

2-Sample t-Test of $\mu_1 - \mu_2$

No Selector

Individual Alpha Level 0.05

Ho: $\mu_1 - \mu_2 = 0$ Ha: $\mu_1 - \mu_2 \neq 0$

EXTRINSIC:SCORE - INTRINSIC:SCORE:

Test Ho: $\mu(\text{EXTRINSIC:SCORE}) - \mu(\text{INTRINSIC:SCORE}) = 0$ vs Ha: $\mu(\text{EXTRINSIC:SCORE}) - \mu(\text{INTRINSIC:SCORE}) \neq 0$

Difference Between Means = -4.1442029 t-Statistic = -2.915 w/43 df

Reject Ho at Alpha = 0.05

p = 0.0056

Pulse Rate Example

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Does regular exercise reduce resting pulse rates? 10 volunteers do 20 minutes of exercise 3 times a week for 6 weeks. Their resting pulse rates before and after were measured.
(Beats/min.)

Pulse Rate Example

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Subject	Before	After
Allen	73	73
Brandon	83	79
Carlos	85	81
David	87	86
Edwin	91	87
Franco	99	91
Graeme	87	84
Hans	85	83
Ivan	83	84
Jorge	79	76

Pulse Rate Example

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Subject	Before	After
Allen	73	73
Brandon	83	79
Carlos	85	81
David	87	86
Edwin	91	87
Franco	99	91
Graeme	87	84
Hans	85	83
Ivan	83	84
Jorge	79	76

Notice that 8 out of 10 subjects had reduced RPR's. There is a non-parametric test called the sign test that could be used to conclude that RPR's tend to decrease as hypothesized. **Can you see how binomial distribution ideas could be used for this?**

Pulse Rate Example

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Notation: Let μ_b be the mean resting pulse rate before the exercise program and μ_a the mean resting pulse rate after.

Pulse Rate Example

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Notation: Let μ_b be the mean resting pulse rate before the exercise program and μ_a the mean resting pulse rate after.

Hypotheses:

- $H_0: \mu_b = \mu_a$ (or $\mu_b - \mu_a \leq 0$ or ...)
- $H_a: \mu_b > \mu_a$

Pulse Rate Example

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Since individuals have important characteristics besides their exercise programs which determine their RPR's, statistical tests based on

$$\text{Var}(\bar{x}_b - \bar{x}_a) = \text{Var}(\bar{x}_b) + \text{Var}(\bar{x}_a)$$

would not be appropriate here.

Pulse Rate Example

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

In MP, we look at the differences and apply 1-sample ideas to the differences.

Pulse Rate Example

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Differences d_i

Subject	Before - After
Allen	0
Brandon	4
Carlos	4
David	1
Edwin	4
Franco	8
Graeme	3
Hans	2
Ivan	-1
Jorge	3

Pulse Rate Example

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

The mean $\bar{d} = 2.8$ with a standard deviation s of 2.8.

Note you need to directly calculate s for the differences; it is not determined by s_a and s_b .

Pulse Rate Example

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

The mean $\bar{d} = 2.8$ with a standard deviation s of 2.8.

$$t = \frac{2.8}{\frac{2.53}{\sqrt{10}}} = -3.50$$

with $df=9$. Our P-value is $< .005$ and we reject H_0 . **Exercise does seem to reduce RPR's.**

Matched Pairs vs. Indep. 2-Sample

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Group1	Group 2	
x_1	y_1	
x_2	y_2	
\dots	\dots	
x_n	\dots	
	y_m	
\bar{x}	\bar{y}	mean
s_x	s_y	std. dev

Matched Pairs vs. Indep. 2-Sample

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Group 1	Group 2	
x_1	y_1	
x_2	y_2	
\dots	\dots	
x_n	\dots	
	y_m	
\bar{x}	\bar{y}	mean
s_x	s_y	std. dev

Independent 2-Sample

Each x_i and each y_j are (except to the extent that they come from one of the groups) independent of each other.

The sample sizes m and n can be different.

Matched Pairs vs. Indep. 2-Sample

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Group1	Group 2	
x_1	y_1	
x_2	y_2	
\dots	\dots	
x_n	\dots	
	y_m	
\bar{x}	\bar{y}	mean
s_x	s_y	std. dev

Matched Pairs

The i 'th observation x_i in group 1 is more closely related (in a relevant way) to the i 'th observation y_i in group 2 than a general x_i is to a general y_j .

The sample sizes m and n must be the same.

Matched Pairs or Indep 2-Sample?

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Chapter 25 # 24: In a test of breaking performance, a tire manufacturer measured the stopping distance for one of its tire models. On a test track, a car made repeated stops from 60 miles per hour. The test was run on both dry and wet pavement, with results as shown in the table. **Are wet stopping distances greater?**

Matched Pairs or Indep 2-Sample?

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Stopping Distance (ft)	
Dry Pavement	Wet Pavement
145	211
152	191
141	220
143	207
131	198
148	208
126	206
140	177
135	183
133	223

Matched Pairs or Indep 2-Sample?

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Chapter 25 # 24: In a test of breaking performance, a tire manufacturer measured the stopping distance for one of its tire models. On a test track, a car made repeated stops from 60 miles per hour. The test was run on both dry and wet pavement, with results as shown in the table. **Are wet stopping distances greater?**

The wet pavement figure on e.g. the fifth line bears no closer relationship to the fifth line in the dry column than it does to any other dry measurement. **There is no matching.** This is independent 2-sample.

Matched Pairs or Indep 2-Sample?

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Had the problem referred to 10 different cars or 10 different drivers each trying both wet and dry, it would have been matched pairs.

Matched Pairs or Indep 2-Sample?

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

	The Data	
	Mean	Std. Dev.
dry	139.4	8.10
wet	202.4	15.07

Matched Pairs or Indep 2-Sample?

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Notation: Let μ_d be the mean dry stopping distance.
and μ_w be the mean wet stopping distance.

Matched Pairs or Indep 2-Sample?

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Notation: Let μ_d be the mean dry stopping distance.
and μ_w be the mean wet stopping distance.

Hypotheses:

- $H_0: \mu_d = \mu_w$ (or $\mu_w - \mu_d \geq 0$ or ...)
- $H_a: \mu_w > \mu_d$

Matched Pairs or Indep 2-Sample?

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Hypotheses:

- $H_0: \mu_d = \mu_w$ (or $\mu_w - \mu_d \geq 0$ or ...)
- $H_a: \mu_w > \mu_d$

$n_w = n_d = 10$, so we use $df = 9$ for hand calculation.

Matched Pairs or Indep 2-Sample?

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Hypotheses:

- $H_0: \mu_d = \mu_w$ (or $\mu_w - \mu_d \geq 0$ or ...)
- $H_a: \mu_w > \mu_d$

$n_w = n_d = 10$, so we use $df = 9$ for hand calculation.

$$SE = \sqrt{\frac{8.10^2}{10} + \frac{15.07^2}{10}} = 5.41$$

Matched Pairs or Indep 2-Sample?

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Hypotheses:

- $H_0: \mu_d = \mu_w$ (or $\mu_w - \mu_d \geq 0$ or ...)
- $H_a: \mu_w > \mu_d$

$n_w = n_d = 10$, so we use $df = 9$ for hand calculation.

$$SE = \sqrt{\frac{8.10^2}{10} + \frac{15.07^2}{10}} = 5.41$$

$$t = \frac{202.4 - 139.4}{5.41} = 11.65.$$

Matched Pairs or Indep 2-Sample?

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Hypotheses:

- $H_0: \mu_d = \mu_w$ (or $\mu_w - \mu_d \geq 0$ or ...)
- $H_a: \mu_w > \mu_d$

$n_w = n_d = 10$, so we use $df = 9$ for hand calculation.

$$SE = \sqrt{\frac{8.10^2}{10} + \frac{15.07^2}{10}} = 5.41$$

$$t = \frac{202.4 - 139.4}{5.41} = 11.65.$$

Our P-value is very tiny ($< .005$ from the tables) so we reject the null; Wet stopping distances appear to be longer.

Matched Pairs or Indep 2-Sample?

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Chapter 25 # 19: A company institutes an exercise break for its workers to see if this will improve job satisfaction, as measured by a questionnaire that assesses workers' satisfaction. Scores of 10 randomly selected workers before and after the implementation of the exercise program are shown.

Matched Pairs or Indep 2-Sample?

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Job Satisfaction Index		
Worker Number	Before	After
1	34	33
2	28	36
3	29	50
4	45	41
5	26	37
6	27	41
7	24	39
8	15	21
9	15	20
10	27	37

Matched Pairs or Indep 2-Sample?

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Chapter 25 # 21: Do these data suggest that there is a significant difference in calories between servings of strawberry and vanilla yogurt?

Matched Pairs or Indep 2-Sample?

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Brand	Calories per Serving	
	Strawberry	Vanilla
America's Choice	210	200
Breyer's Lowfat	220	220
Columbo	220	180
Dannon Light n' Fit	120	120
Dannon Lowfat	210	230
Dannon laCreme	140	140
Great Value	180	80
La Yogurt	170	160
Mountain High	200	170
Stonyfield Farm	100	120
Yoplait Custard	190	190
Yoplait Light	100	100

MP as more Sensitive

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

**MP as more
Sensitive**

t-Conditions

Robustness

Conditions for t Tests

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

- plausible independence
- random sampling
- 10% condition
- nearly normal condition
 - $n < 15$ unimodal and symmetric
 - $15 \leq n \leq 40$ avoid strong skewness and outliers
(unimodal and sym best)
 - $n > 40$ pretty much ok

Conditions for t Tests

- plausible independence
- random sampling
- 10% condition
- nearly normal condition

$n < 15$ unimodal and symmetric

$15 \leq n \leq 40$ avoid strong skewness and outliers
(unimodal and sym best)

$n > 40$ pretty much ok

What Happens if Not Satisfied:

random sampling -

could be critical;

might be ok if "representative"

representative hard/impossible to define

Conditions for t Tests

- plausible independence
- random sampling
- 10% condition
- nearly normal condition

$n < 15$ unimodal and symmetric

$15 \leq n \leq 40$ avoid strong skewness and outliers
(unimodal and sym best)

$n > 40$ pretty much ok

What Happens if Not Satisfied:

plausible independence -
could be critical
sometimes just a working hypothesis

Conditions for t Tests

- plausible independence
- random sampling
- 10% condition
- nearly normal condition

$n < 15$ unimodal and symmetric

$15 \leq n \leq 40$ avoid strong skewness and outliers
(unimodal and sym best)

$n > 40$ pretty much ok

What Happens if Not Satisfied:

10% condition -

results in overestimation of samp. dist. st dev
gradual breakdown in formulas, not method

Conditions for t Tests

- plausible independence
- random sampling
- 10% condition
- nearly normal condition

$n < 15$ unimodal and symmetric

$15 \leq n \leq 40$ avoid strong skewness and outliers
(unimodal and sym best)

$n > 40$ pretty much ok

What Happens if Not Satisfied:

nearly normal -

no guarantee

progressive reduction of accuracy

Conditions for t Tests

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

For 2-sample inference, we add the

independence groups assumption.

The chance of an individual in one of the groups assuming a certain value should be independent of the values assumed by any of the individuals in the other group.

Robustness

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Formulas for t-inference and regression inference are based on assumptions of normality of the data.

Yet most distributions are not normal. (Although the CLT makes averages of lots normal.)

So it is perhaps remarkable that t-inference methods for moderate size data sets without outliers are typically pretty good. **Why is this?**

Robustness

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Formulas for t-inference and regression inference are based on assumptions of normality of the data.

Yet most distributions are not normal. (Although the CLT makes averages of lots normal.)

So it is perhaps remarkable that t-inference methods for moderate size data sets without outliers are typically pretty good. **Why is this?**

One answer is robustness against non-normality.

Robustness

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

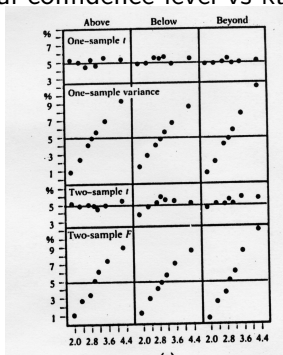
Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Actual confidence level vs kurtosis



for some symmetric $n = 25$ test distributions in a 1975 Biometrika paper of Pearson and Please. (kurtosis=3 in the normal case.)

Robustness

Math 1710
Class 33

V2

Using Table T

A Sample Size
Problem

An Indep
2-Sample
Example

df and
Satterthwaite

Data Desk
View of
Creativity

Matched Pairs

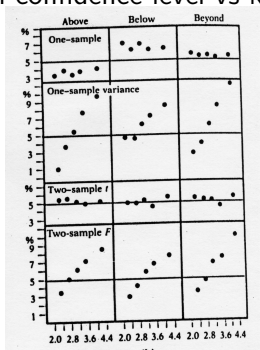
Indep.
2-Sample vs
MP Examples

MP as more
Sensitive

t-Conditions

Robustness

Actual confidence level vs kurtosis



for some skewed $n = 25$ test distributions in a 1975 *Biometrika* paper of Pearson and Please.