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Matched Pairs

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MP as more
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Using Table χ

Chi Square
Distribution as
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Math 1710 Class 34

2-Sample and MP, Intro to Chi Square
Dr. Back

Nov. 16, 2009

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Summary of **INTRINSIC:SCORE**
No Selector

Percentile 25

Count	24
Mean	19.8833
Median	20.4
StdDev	4.43951
Range	17.7
IntQRange	5.05
Lower ith %tile	17.35
Upper ith %tile	22.4

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Summary of **EXTRINSIC:SCORE**
No Selector

Percentile 25

Count	23
Mean	15.7391
Median	17.2
StdDev	5.2526
Range	19
IntQRange	7
Lower ith %tile	12.075
Upper ith %tile	19.075

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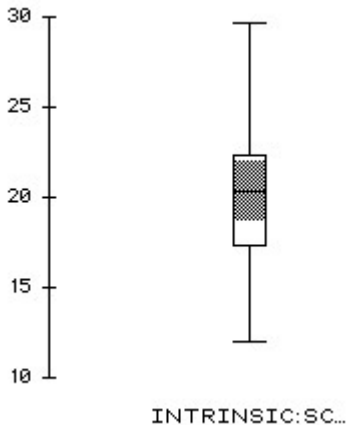
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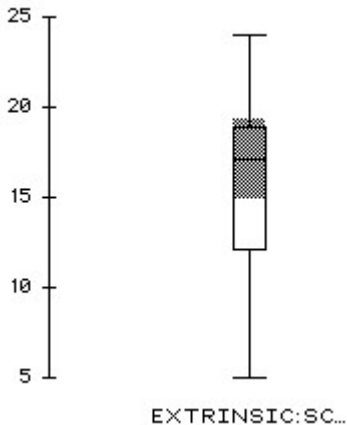
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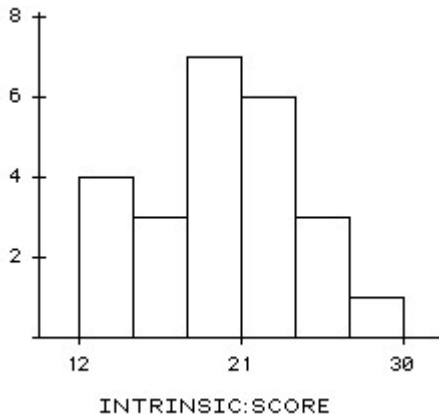
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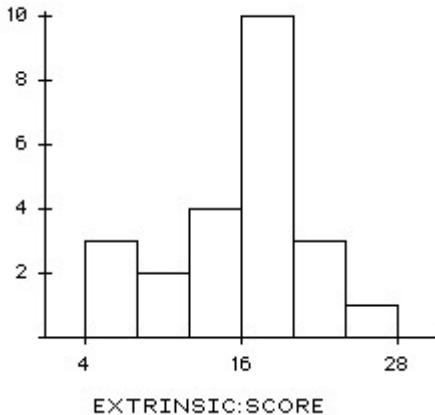
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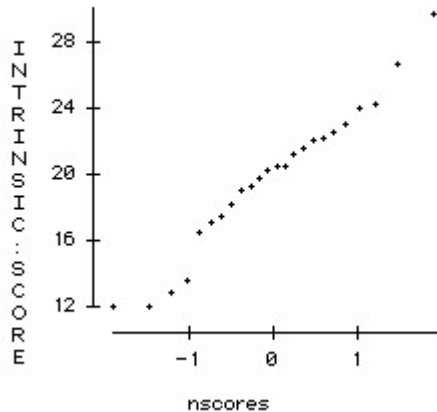
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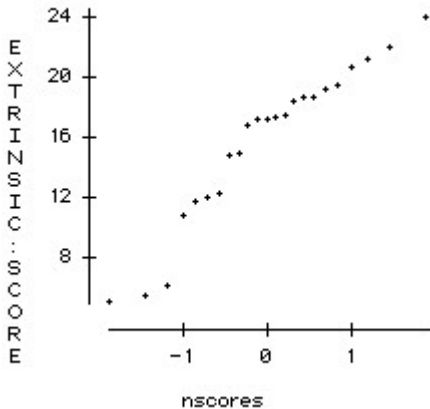
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t-Interval for Individual μ 's

No Selector

Individual Confidence 95.00%

Bounds: Lower Bound $< \mu <$ Upper Bound

With 95.00% Confidence, 18.008691 $< \mu$ (INTRINSIC:SCORE) $<$ 21.757975

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t-Interval for Individual μ 's

No Selector

Individual Confidence 95.00%

Bounds: Lower Bound $< \mu <$ Upper Bound

With 95.00% Confidence, $13.467738 < \mu(\text{EXTRINSIC:SCORE}) < 18.010523$

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2-Sample t-Interval for $\mu_1 - \mu_2$

No Selector

Individual Confidence 95.00%

Bounds: Lower Bound $< \mu_1 - \mu_2 <$ Upper Bound

With 95.00% Confidence, $-7.0108029 < \mu(\text{EXTRINSIC:SCORE}) - \mu(\text{INTRINSIC:SCORE}) < -1.2776029$

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2-Sample t-Test of $\mu_1 - \mu_2$

No Selector

Individual Alpha Level 0.05

Ho: $\mu_1 - \mu_2 = 0$ Ha: $\mu_1 - \mu_2 \neq 0$

EXTRINSIC:SCORE - INTRINSIC:SCORE :

Test Ho: $\mu(\text{EXTRINSIC:SCORE}) - \mu(\text{INTRINSIC:SCORE}) = 0$ vs Ha: $\mu(\text{EXTRINSIC:SCORE}) - \mu(\text{INTRINSIC:SCORE}) \neq 0$

Difference Between Means = -4.1442029 t-Statistic = -2.915 w/43 df

Reject Ho at Alpha = 0.05

p = 0.0056

Pulse Rate Example

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Does regular exercise reduce resting pulse rates? 10 volunteers do 20 minutes of exercise 3 times a week for 6 weeks. Their resting pulse rates before and after were measured.
(Beats/min.)

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Subject	Before	After
Allen	73	73
Brandon	83	79
Carlos	85	81
David	87	86
Edwin	91	87
Franco	99	91
Graeme	87	84
Hans	85	83
Ivan	83	84
Jorge	79	76

Pulse Rate Example

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Subject	Before	After
Allen	73	73
Brandon	83	79
Carlos	85	81
David	87	86
Edwin	91	87
Franco	99	91
Graeme	87	84
Hans	85	83
Ivan	83	84
Jorge	79	76

Notice that 8 out of 10 subjects had reduced RPR's. There is a non-parametric test called the sign test that could be used to conclude that RPR's tend to decrease as hypothesized. **Can you see how binomial distribution ideas could be used for this?**

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Notation: Let μ_b be the mean resting pulse rate before the exercise program and μ_a the mean resting pulse rate after.

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Notation: Let μ_b be the mean resting pulse rate before the exercise program and μ_a the mean resting pulse rate after.

Hypotheses:

- $H_0: \mu_b = \mu_a$ (or $\mu_b - \mu_a \leq 0$ or ...)
- $H_a: \mu_b > \mu_a$

Pulse Rate Example

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Since individuals have important characteristics besides their exercise programs which determine their RPR's, statistical tests based on

$$\text{Var}(\bar{x}_b - \bar{x}_a) = \text{Var}(\bar{x}_b) + \text{Var}(\bar{x}_a)$$

would not be appropriate here.

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In MP, we look at the differences and apply 1-sample ideas to the differences.

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Differences d_i

Subject	Before - After
Allen	0
Brandon	4
Carlos	4
David	1
Edwin	4
Franco	8
Graeme	3
Hans	2
Ivan	-1
Jorge	3

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The mean $\bar{d} = 2.8$ with a standard deviation s of 2.8.

Note you need to directly calculate s for the differences; it is not determined by s_a and s_b .

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The mean $\bar{d} = 2.8$ with a standard deviation s of 2.8.

$$t = \frac{2.8}{\frac{2.53}{\sqrt{10}}} = -3.50$$

with $df=9$. Our P-value is $< .005$ and we reject H_0 . **Exercise does seem to reduce RPR's.**

Matched Pairs vs. Indep. 2-Sample

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Group1	Group 2	
x_1	y_1	
x_2	y_2	
\dots	\dots	
x_n	\dots	
	y_m	
\bar{x}	\bar{y}	mean
s_x	s_y	std. dev

Matched Pairs vs. Indep. 2-Sample

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Group1	Group 2	
x_1	y_1	
x_2	y_2	
\dots	\dots	
x_n	\dots	
	y_m	
\bar{x}	\bar{y}	mean
s_x	s_y	std. dev

Independent 2-Sample

Each x_i and each y_j are (except to the extent that they come from one of the groups) independent of each other.

The sample sizes m and n can be different.

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Group1	Group 2	
x_1	y_1	
x_2	y_2	
\dots	\dots	
x_n	\dots	
	y_m	
\bar{x}	\bar{y}	mean
s_x	s_y	std. dev

Matched Pairs

The i 'th observation x_i in group 1 is more closely related (in a relevant way) to the i 'th observation y_i in group 2 than a general x_i is to a general y_j .

The sample sizes m and n must be the same.

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Chapter 25 # 24: In a test of breaking performance, a tire manufacturer measured the stopping distance for one of its tire models. On a test track, a car made repeated stops from 60 miles per hour. The test was run on both dry and wet pavement, with results as shown in the table. **Are wet stopping distances greater?**

Matched Pairs or Indep 2-Sample?

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Stopping Distance (ft)	
Dry Pavement	Wet Pavement
145	211
152	191
141	220
143	207
131	198
148	208
126	206
140	177
135	183
133	223

Matched Pairs or Indep 2-Sample?

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Chapter 25 # 24: In a test of breaking performance, a tire manufacturer measured the stopping distance for one of its tire models. On a test track, a car made repeated stops from 60 miles per hour. The test was run on both dry and wet pavement, with results as shown in the table. **Are wet stopping distances greater?**

The wet pavement figure on e.g. the fifth line bears no closer relationship to the fifth line in the dry column than it does to any other dry measurement. **There is no matching.** This is independent 2-sample.

Matched Pairs or Indep 2-Sample?

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Had the problem referred to 10 different cars or 10 different drivers each trying both wet and dry, it would have been matched pairs.

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Summary of the Data

	Mean	Std. Dev.
dry	139.4	8.10
wet	202.4	15.07

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Notation: Let μ_d be the mean dry stopping distance.
and μ_w be the mean wet stopping distance.

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Notation: Let μ_d be the mean dry stopping distance.
and μ_w be the mean wet stopping distance.

Hypotheses:

- $H_0: \mu_d = \mu_w$ (or $\mu_w - \mu_d \geq 0$ or ...)
- $H_a: \mu_w > \mu_d$

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Hypotheses:

- $H_0: \mu_d = \mu_w$ (or $\mu_w - \mu_d \geq 0$ or ...)
- $H_a: \mu_w > \mu_d$

$n_w = n_d = 10$, so we use $df = 9$ for hand calculation.

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Hypotheses:

- $H_0: \mu_d = \mu_w$ (or $\mu_w - \mu_d \geq 0$ or ...)
- $H_a: \mu_w > \mu_d$

$n_w = n_d = 10$, so we use $df = 9$ for hand calculation.

$$SE = \sqrt{\frac{8.10^2}{10} + \frac{15.07^2}{10}} = 5.41$$

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Hypotheses:

- $H_0: \mu_d = \mu_w$ (or $\mu_w - \mu_d \geq 0$ or ...)
- $H_a: \mu_w > \mu_d$

$n_w = n_d = 10$, so we use $df = 9$ for hand calculation.

$$SE = \sqrt{\frac{8.10^2}{10} + \frac{15.07^2}{10}} = 5.41$$

$$t = \frac{202.4 - 139.4}{5.41} = 11.65.$$

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Hypotheses:

- $H_0: \mu_d = \mu_w$ (or $\mu_w - \mu_d \geq 0$ or ...)
- $H_a: \mu_w > \mu_d$

$n_w = n_d = 10$, so we use $df = 9$ for hand calculation.

$$SE = \sqrt{\frac{8.10^2}{10} + \frac{15.07^2}{10}} = 5.41$$

$$t = \frac{202.4 - 139.4}{5.41} = 11.65.$$

Our P-value is very tiny ($< .005$ from the tables) so we reject the null; Wet stopping distances appear to be longer.

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Chapter 25 # 19: A company institutes an exercise break for its workers to see if this will improve job satisfaction, as measured by a questionnaire that assesses workers' satisfaction. Scores of 10 randomly selected workers before and after the implementation of the exercise program are shown.

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Job Satisfaction Index		
Worker Number	Before	After
1	34	33
2	28	36
3	29	50
4	45	41
5	26	37
6	27	41
7	24	39
8	15	21
9	15	20
10	27	37

Matched Pairs or Indep 2-Sample?

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Worker (e.g.) 5's job satisfaction after the program clearly has some extra association with his job satisfaction before. We have a clear matching here and MP is appropriate.

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Summary of the Data

	Mean	Std. Dev.
before -after	-8.50	7.47

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Notation: Let μ_b be the mean job satisfaction before.
and μ_a be the mean job satisfaction after.

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Notation: Let μ_b be the mean job satisfaction before.
and μ_a be the mean job satisfaction after.

Hypotheses:

- $H_0: \mu_b = \mu_a$
- $H_a: \mu_a > \mu_b$

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Hypotheses:

- $H_0: \mu_b = \mu_a$
- $H_a: \mu_a > \mu_b$

$n_b = n_a = 10$, so we use $df = 9$ for hand calculation.

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Hypotheses:

- $H_0: \mu_b = \mu_a$

- $H_a: \mu_a > \mu_b$

$n_b = n_a = 10$, so we use $df = 9$ for hand calculation.

$$SE = \frac{s}{\sqrt{n}} = \frac{7.47}{\sqrt{10}} = 2.36$$

Matched Pairs or Indep 2-Sample?

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an RV

Hypotheses:

- $H_0: \mu_b = \mu_a$

- $H_a: \mu_a > \mu_b$

$n_b = n_a = 10$, so we use $df = 9$ for hand calculation.

$$SE = \frac{s}{\sqrt{n}} = \frac{7.47}{\sqrt{10}} = 2.36$$

$$t = \frac{-8.5}{2.36} = 3.598.$$

Matched Pairs or Indep 2-Sample?

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Hypotheses:

- $H_0: \mu_b = \mu_a$
- $H_a: \mu_a > \mu_b$

$n_b = n_a = 10$, so we use $df = 9$ for hand calculation.

$$SE = \frac{s}{\sqrt{n}} = \frac{7.47}{\sqrt{10}} = 2.36$$

$$t = \frac{-8.5}{2.36} = 3.598.$$

Our P-value is very tiny ($< .005$ from the tables) so we reject the null; The program appears to improve job satisfaction.

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Chapter 25 # 21: Do these data suggest that there is a significant difference in calories between servings of strawberry and vanilla yogurt?

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Brand	Calories per Serving	
	Strawberry	Vanilla
America's Choice	210	200
Breyer's Lowfat	220	220
Columbo	220	180
Dannon Light n' Fit	120	120
Dannon Lowfat	210	230
Dannon laCreme	140	140
Great Value	180	80
La Yogurt	170	160
Mountain High	200	170
Stonyfield Farm	100	120
Yoplait Custard	190	190
Yoplait Light	100	100

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Since the two flavors are the same brand, MP is appropriate.
There is also an outlier Great Value which we will exclude from our analysis.

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Summary of the Data (w/o Great Value)

	Mean	Std. Dev.
strawberry-vanilla	4.55	18.09

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Notation: Let μ_s be the mean calories in a strawberry serving.
and μ_v be the mean calories in a vanilla serving.

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Notation: Let μ_s be the mean calories in a strawberry serving.
and μ_v be the mean calories in a vanilla serving.

Hypotheses:

- $H_0: \mu_s = \mu_v$
- $H_a: \mu_s \neq \mu_v$

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Hypotheses:

- $H_0: \mu_s = \mu_v$
- $H_a: \mu_s \neq \mu_v$

$n_s = n_v = 11$, so we use $df = 10$ for hand calculation.

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Hypotheses:

- $H_0: \mu_s = \mu_v$
- $H_a: \mu_s \neq \mu_v$

$n_s = n_v = 11$, so we use $df = 10$ for hand calculation.

$$SE = \frac{s}{\sqrt{n}} = \frac{18.09}{\sqrt{11}} = 5.45$$

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Hypotheses:

- $H_0: \mu_s = \mu_v$
- $H_a: \mu_s \neq \mu_v$

$n_s = n_v = 11$, so we use $df = 10$ for hand calculation.

$$SE = \frac{s}{\sqrt{n}} = \frac{18.09}{\sqrt{11}} = 5.45$$

$$t = \frac{4.55}{5.45} = .835$$

Matched Pairs or Indep 2-Sample?

Hypotheses:

- $H_0: \mu_s = \mu_v$
- $H_a: \mu_s \neq \mu_v$

$n_s = n_v = 11$, so we use $df = 10$ for hand calculation.

$$SE = \frac{s}{\sqrt{n}} = \frac{18.09}{\sqrt{11}} = 5.45$$

$$t = \frac{4.55}{5.45} = .835$$

Our tail probability is above .1 from the tables and so our P-value is above .2. We retain the null. We haven't proven there are any differences in calories between strawberry and vanilla of a give brand.

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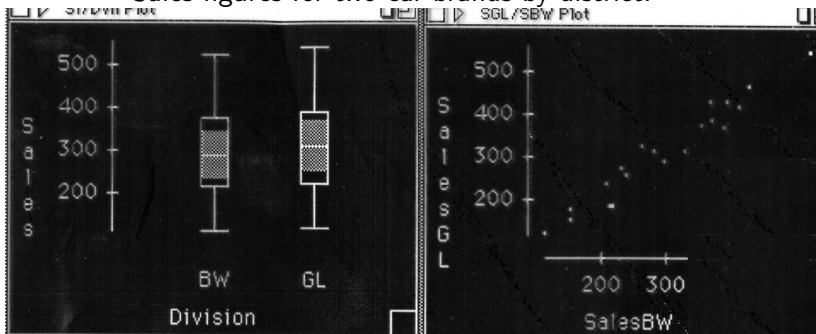
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Sales figures for two car brands by district.



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test

2-Sample t-Test of $\mu_1 - \mu_2$

No Selector

Individual Alpha Level 0.05

Ho: $\mu_1 - \mu_2 = 0$ Ha: $\mu_1 - \mu_2 \neq 0$

Hide Results

SalesGL - SalesBW :

Test Ho: $\mu(\text{SalesGL}) - \mu(\text{SalesBW}) = 0$ vs Ha: $\mu(\text{SalesGL}) - \mu(\text{SalesBW}) \neq 0$

Difference Between Means = 13.500000 t-Statistic = 0.3

Fail to reject Ho at Alpha = 0.05

p = 0.7058

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The screenshot shows a Minitab Test dialog box for a Paired t-Test. The title bar reads "Test". The main title is "Paired t-Test of $\mu(1 - 2)$ ". Under "No Selector", the "Individual" radio button is selected, and the "Alpha Level" is set to 0.05. The null hypothesis is $H_0: \mu(1 - 2) = 0$ and the alternative hypothesis is $H_a: \mu(1 - 2) \neq 0$. A "Hide Results" button is visible. The test results are displayed below: "Sales6L - Sales6W : Test $H_0: \mu(\text{Sales6L} - \text{Sales6W}) = 0$ vs $H_a: \mu(\text{Sales6L} - \text{Sales6W}) \neq 0$. Mean of Paired Differences = 13.500000 t-Statistic = 2.257 w/19 df. Reject H_0 at Alpha = 0.05. p = 0.0360".

Test

Paired t-Test of $\mu(1 - 2)$

No Selector

Individual Alpha Level 0.05

$H_0: \mu(1 - 2) = 0$ $H_a: \mu(1 - 2) \neq 0$

Hide Results

Sales6L - Sales6W :

Test $H_0: \mu(\text{Sales6L} - \text{Sales6W}) = 0$ vs $H_a: \mu(\text{Sales6L} - \text{Sales6W}) \neq 0$

Mean of Paired Differences = 13.500000 t-Statistic = 2.257 w/19 df

Reject H_0 at Alpha = 0.05

p = 0.0360

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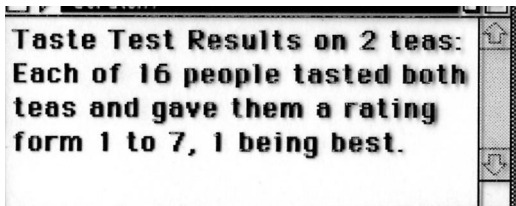
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**Taste Test Results on 2 teas:
Each of 16 people tasted both
teas and gave them a rating
form 1 to 7, 1 being best.**

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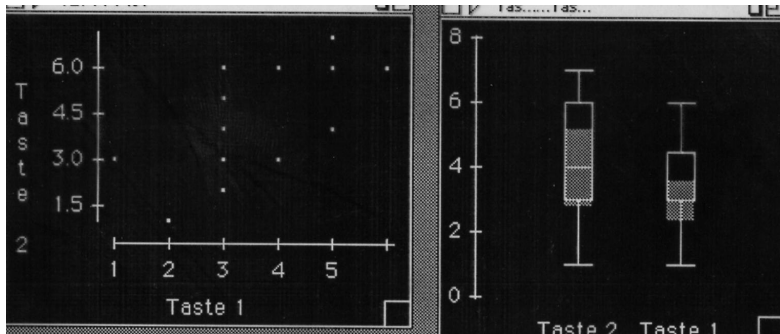
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**MP as more
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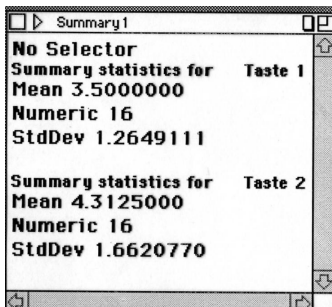
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The screenshot shows a window titled "Summary1" with the following content:

No Selector	
Summary statistics for	Taste 1
Mean 3.500000	
Numeric 16	
StdDev 1.2649111	
Summary statistics for	Taste 2
Mean 4.312500	
Numeric 16	
StdDev 1.6620770	

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The screenshot shows a window titled "Estimate" with the following content:

- t-Interval for Individual μ 's**
- No Selector**
- Individual** Confidence **95.00%**
- Bounds:** **Lower Bound > μ < Upper Bound**
- Hide Results**
- With 95.00% Confidence, $3.4268416 < \mu(\text{Taste 2}) < 5.1981584$**
- With 95.00% Confidence, $2.8259765 < \mu(\text{Taste 1}) < 4.1740235$**

MP as more Sensitive

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Paired t-Interval for $\mu(1 - 2)$

No Selector

Individual Confidence 95.00%

Bounds: Lower Bound $> \mu(1 - 2) <$ Upper Bound

Hide Results

With 95.00% Confidence, $0.10507624 < \mu(\text{Taste 2-Taste 1}) < 1.5199238$

Conditions for t Tests

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Chi Square
Distribution as
an RV

- plausible independence
- random sampling
- 10% condition
- nearly normal condition
 - $n < 15$ unimodal and symmetric
 - $15 \leq n \leq 40$ avoid strong skewness and outliers
(unimodal and sym best)
 - $n > 40$ pretty much ok

Conditions for t Tests

- plausible independence
- random sampling
- 10% condition
- nearly normal condition
 - $n < 15$ unimodal and symmetric
 - $15 \leq n \leq 40$ avoid strong skewness and outliers (unimodal and sym best)
 - $n > 40$ pretty much ok

What Happens if Not Satisfied:

random sampling -
could be critical;
might be ok if "representative"
representative hard/impossible to define

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Conditions for t Tests

- plausible independence
- random sampling
- 10% condition
- nearly normal condition

$n < 15$ unimodal and symmetric

$15 \leq n \leq 40$ avoid strong skewness and outliers
(unimodal and sym best)

$n > 40$ pretty much ok

What Happens if Not Satisfied:

plausible independence -

could be critical

sometimes just a working hypothesis

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Conditions for t Tests

- plausible independence
- random sampling
- 10% condition
- nearly normal condition

$n < 15$ unimodal and symmetric

$15 \leq n \leq 40$ avoid strong skewness and outliers
(unimodal and sym best)

$n > 40$ pretty much ok

What Happens if Not Satisfied:

10% condition -

results in overestimation of samp. dist. st dev
gradual breakdown in formulas, not method

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Conditions for t Tests

- plausible independence
- random sampling
- 10% condition
- nearly normal condition
 - $n < 15$ unimodal and symmetric
 - $15 \leq n \leq 40$ avoid strong skewness and outliers (unimodal and sym best)
 - $n > 40$ pretty much ok

What Happens if Not Satisfied:

nearly normal -
no guarantee
progressive reduction of accuracy

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For 2-sample inference, we add the

independence groups assumption.

The chance of an individual in one of the groups assuming a certain value should be independent of the values assumed by any of the individuals in the other group.

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Distribution as
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Formulas for t-inference and regression inference are based on assumptions of normality of the data.

Yet most distributions are not normal. (Although the CLT makes averages of lots normal.)

So it is perhaps remarkable that t-inference methods for moderate size data sets without outliers are typically pretty good. **Why is this?**

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Formulas for t-inference and regression inference are based on assumptions of normality of the data.

Yet most distributions are not normal. (Although the CLT makes averages of lots normal.)

So it is perhaps remarkable that t-inference methods for moderate size data sets without outliers are typically pretty good. **Why is this?**

One answer is robustness against non-normality.

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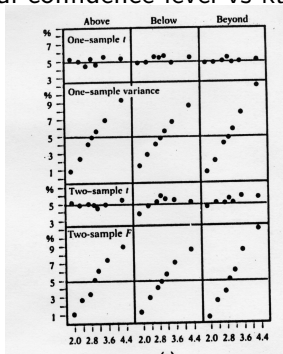
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Actual confidence level vs kurtosis



for some symmetric $n = 25$ test distributions in a 1975 Biometrika paper of Pearson and Please. (kurtosis=3 in the normal case.)

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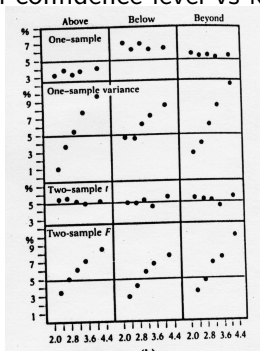
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Actual confidence level vs kurtosis



for some skewed $n = 25$ test distributions in a 1975 *Biometrika* paper of Pearson and Please.

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Flu Shots (categorical) vs Getting the Flu

	0-shot	1-shot	2-shot
flu	24	9	13
no flu	289	100	565

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With Marginals

	0-shot	1-shot	2-shot	
flu	24	9	13	46
no flu	289	100	565	954
	313	109	578	1000

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With Marginals

	0-shot	1-shot	2-shot	
flu	24	9	13	46
no flu	289	100	565	954
	313	109	578	1000

If independent, the expected number of 0-shot people getting the flu would be

$$\begin{aligned} & \text{overall fraction getting flu} \times \text{number 0-shot people} \\ &= \frac{46}{1000} \times 313 = 14.4 \end{aligned}$$

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Chi Square Test of Independence

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If independent, the expected number of 0-shot people getting the flu would also be

$$\begin{aligned} & \text{overall fraction getting 0-shots} \times \text{number getting flu} \\ &= \frac{313}{1000} \times 46 = 14.4 \end{aligned}$$

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If independent, the expected number of 0-shot people getting the flu would be

$$\begin{aligned} \text{overall fraction getting flu} \times \text{number 0-shot people} \\ = \frac{46}{1000} \times 313 = 14.4 \end{aligned}$$

This comes down to

$$\text{expected value} = \frac{\text{row sum} \times \text{col sum}}{\text{total}}$$

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With Expected Values

	0-shot	1-shot	2-shot	
flu	24 (14.4)	9 (5.0)	13 (26.6)	46
no flu	289 (298.6)	100 (104)	565 (551.4)	954
	313	109	578	1000

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	0-shot	1-shot	2-shot	
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no flu	289 (298.6)	100 (104)	565 (551.4)	954
	313	109	578	1000

$$\chi^2 = \sum \frac{(Obs - Exp)^2}{Exp}$$

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no flu	289 (298.6)	100 (104)	565 (551.4)	954
	313	109	578	1000

$$\chi^2 = \sum \frac{(Obs - Exp)^2}{Exp} = \frac{(24 - 14.4)^2}{14.4} + \dots \text{(5 more terms)}$$

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	0-shot	1-shot	2-shot	
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no flu	289 (298.6)	100 (104)	565 (551.4)	954
	313	109	578	1000

$$\chi^2 = \sum \frac{(Obs - Exp)^2}{Exp} = \frac{(24 - 14.4)^2}{14.4} + \dots = 17.35$$

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no flu	289 (298.6)	100 (104)	565 (551.4)	954
	313	109	578	1000

$$\chi^2 = \sum \frac{(Obs - Exp)^2}{Exp} = \frac{(24 - 14.4)^2}{14.4} + \dots = 17.35$$

χ square tests also have a degrees of freedom.

For a test of independence, the number of rows (nr) and the number of columns (nc) in the original contingency table are the numbers of categories of the resp. cat. vars.

$$df = (nr - 1)(nc - 1)$$

Chi Square Test of Independence

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Chi Square

Using Table χ

Chi Square
Distribution as
an RV

With Expected Values

	0-shot	1-shot	2-shot	
flu	24 (14.4)	9 (5.0)	13 (26.6)	46
no flu	289 (298.6)	100 (104)	565 (551.4)	954
	313	109	578	1000

$$\chi^2 = \sum \frac{(Obs - Exp)^2}{Exp} = \frac{(24 - 14.4)^2}{14.4} + \dots = 17.35$$

$$df = (nr - 1)(nc - 1)$$

Here

$$df = (2 - 1)(3 - 1) = 2.$$

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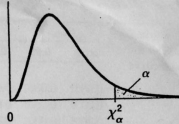
Chi Square

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Right tail probability

Table X
Values of χ^2_α



	0.10	0.05	0.025	0.01	0.005
df					
1	2.706	3.841	5.024	6.635	7.879
2	4.605	5.991	7.378	9.210	10.597
3	6.251	7.815	9.348	11.345	12.838
4	7.779	9.488	11.143	13.277	14.860
...					
15	22.307	24.996	27.488	30.578	32.801
16	23.542	26.296	28.845	32.000	34.267
17	24.769	27.587	30.191	33.409	35.718
18	25.989	28.869	31.526	34.805	37.156
19	27.204	30.143	32.852	36.191	38.582

These are all critical values χ^* .

Using Table χ

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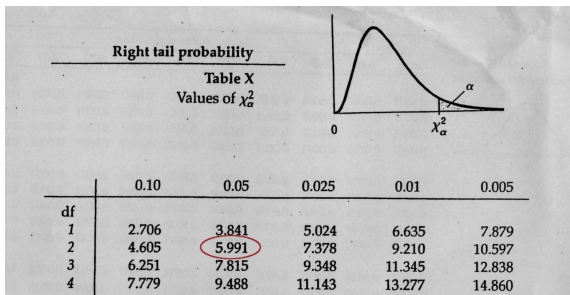
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...

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These are all critical values χ^* .

For example $P(\chi > 5.991) = .05$ for the χ square distribution with 2 df.

Using Table χ

Our χ square-statistic of 17.35 is more extreme than any on the $df=2$ row of the table.

The picture



shows what the critical value $\chi^* = 10.597$ for a tail prob. of $.005$ means.

Using Table χ

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So our tail probability and P-value are both less than .005 and we reject the null. Flu shots are making a difference.

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What we just did is actually a hypothesis test.

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Hypotheses:

- H_0 : The categorical variables FluShot? and GetTheFlu? are independent.
- H_a : The categorical variables are not independent. (*i.e. there is an association.*)

Chi Square Distribution as an RV

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Let X_1, \dots, X_d be d independent standard normal random variables.

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Chi Square
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Let X_1, \dots, X_d be d independent standard normal random variables.

The **Chi Square distribution with d degrees of freedom** is given by:

$$\chi^2 = X_1^2 + X_2^2 + \dots + X_d^2$$

Chi Square Distribution as an RV

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Chi Square Distribution Formula

$$f(x) = \frac{1}{\Gamma(\frac{d}{2})2^{\frac{d}{2}}} x^{\frac{d}{2}-1} e^{-\frac{x}{2}}$$

where d is the number of degrees of freedom.

Chi Square Distribution as an RV

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where d is the number of degrees of freedom.

The mean is d and the variance $2d$.