

Math 1710 Class 28

HT's, Type I/II Errors, and Power
Dr. Back

Nov. 2, 2009

Smoke Detectors

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

CPSC '96: 90% of American homes have at least one smoke detector. After a public safety campaign, a city observes that 376 out of 400 randomly selected homes have a detector.

Smoke Detectors

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

CPSC '96: 90% of American homes have at least one smoke detector. After a public safety campaign, a city observes that 376 out of 400 randomly selected homes have a detector.

Is this strong evidence the local rate is greater than the national rate?

Smoke Detectors

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

CPSC '96: 90% of American homes have at least one smoke detector. After a public safety campaign, a city observes that 376 out of 400 randomly selected homes have a detector.

Is this strong evidence the local rate is greater than the national rate?

$$\text{So } \hat{p} = \frac{376}{400} = .94.$$

HT Vocabulary List

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

- Null Hypothesis H_0
- Alternative Hypothesis H_a
(One Sided vs Two Sided)
- $SD(\hat{p})$ vs. $SE(\hat{p})$
- z-statistic
- Tail Probability
- P-value
- Significance Level
- Retain Null H_0 vs. Accept Alt H_a (equiv. reject Null H_0)

HT Vocabulary Details

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

Null Hypothesis H_0 - retained unless disproven.

HT Vocabulary Details

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

Null Hypothesis H_0 - retained unless disproven.

So when it is not clear which hypothesis is likely true, one retains the null.

HT Vocabulary Details

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

Null Hypothesis H_0 - retained unless disproven.

So when it is not clear which hypothesis is likely true, one retains the null.

Our textbook will always write this as $p = p_0$. Here p_0 is some particular numerical value that we are interested in. Perhaps the historical value of the proportion we are now studying.

HT Vocabulary Details

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

Null Hypothesis H_0 - retained unless disproven.

So when it is not clear which hypothesis is likely true, one retains the null.

Our textbook will always write this as $p = p_0$. Here p_0 is some particular numerical value that we are interested in. Perhaps the historical value of the proportion we are now studying.

Or maybe a specification.

HT Vocabulary Details

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

Null Hypothesis H_0 - retained unless disproven.

So when it is not clear which hypothesis is likely true, one retains the null.

Our textbook will always write this as $p = p_0$. Here p_0 is some particular numerical value that we are interested in. Perhaps the historical value of the proportion we are now studying.

Or maybe a specification.

In the 1-sided case, many would write $p \leq p_0$ or $p \geq p_0$ for H_0 .

HT Vocabulary Details

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

Alternative Hypothesis H_A - Only thing which possibly can be proven in the HT.

HT Vocabulary Details

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

Alternative Hypothesis H_A - Only thing which possibly can be proven in the HT.

The only two-sided possibility is $p \neq p_0$.

Typical verbal form: “strong evidence ... of a change ...”

HT Vocabulary Details

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

Alternative Hypothesis H_A - Only thing which possibly can be proven in the HT.

The only two-sided possibility is $p \neq p_0$.

Typical verbal form: “strong evidence ... of a change ...”

Two one-sided possibilities:

- $p < p_0$

- $p > p_0$

Typical verbal form: “strong evidence ... of an increase ...”

HT Vocabulary Details

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

Alternative Hypothesis H_A - Only thing which possibly can be proven in the HT.

The only two-sided possibility is $p \neq p_0$.

Typical verbal form: “strong evidence ... of a change ...”

Two one-sided possibilities:

- $p < p_0$

- $p > p_0$

Typical verbal form: “strong evidence ... of an increase ...”

So accepting the alternative means a statistically significant result; something was proven.

Retaining the null just means the evidence was not strong enough to convincingly prove the alternative.

HT Vocabulary Details

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

z-statistic - the z-score of \hat{p} in relation to the sampling distribution of \hat{p} *assuming H_0 is true.*

HT Vocabulary Details

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

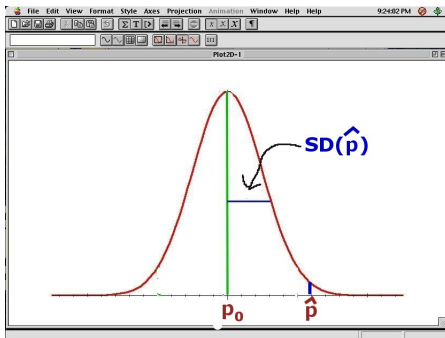
Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

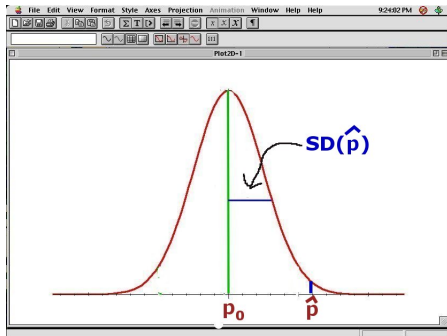
Proportion
Conditions

z-statistic - the z-score of \hat{p} in relation to the sampling distribution of \hat{p} assuming H_0 is true.



HT Vocabulary Details

z-statistic - the z-score of \hat{p} in relation to the sampling distribution of \hat{p} assuming H_0 is true.



$$z = \frac{\hat{p} - p_0}{SD(\hat{p})}$$

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

HT Vocabulary Details

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

z-statistic - the z-score of \hat{p} in relation to the sampling distribution of \hat{p} *assuming* H_0 is true.

$$z = \frac{\hat{p} - p_0}{SD(\hat{p})}$$

“... assuming H_0 is true” is why we use $SD(\hat{p})$ rather than $SE(\hat{p})$ in calculating our z-statistic.

HT Vocabulary Details

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

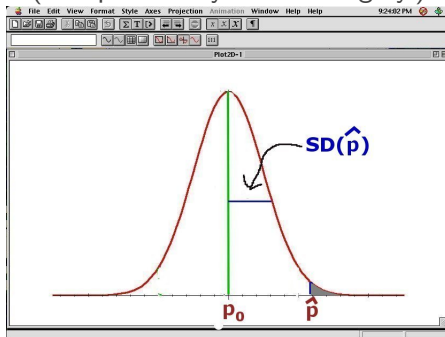
Proportion
Conditions

Tail Probability - The probability of a \hat{p} as extreme or more so **and on the same side of p_0** than the \hat{p} we actually observed. This is again assuming H_0 is true.

HT Vocabulary Details

Tail Probability - The probability of a \hat{p} as extreme or more so **and on the same side of p_0** than the \hat{p} we actually observed. This is again assuming H_0 is true.

(Tail probability shaded in gray.)



Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

HT Vocabulary Details

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

P-value: The probability of a z-statistic as extreme as the given one or more so (and in support of the alternative hypothesis) under the assumption that the null hypothesis is true.

HT Vocabulary Details

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

P-value: The probability of a z-statistic as extreme as the given one or more so (and in support of the alternative hypothesis) under the assumption that the null hypothesis is true.

So P-value =

(single) tail probability in 1-sided test case

twice the (single) tail probability in 2-sided test case.

HT Vocabulary Details

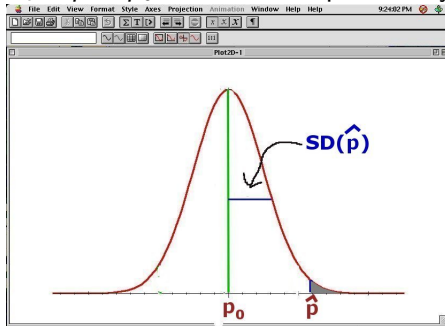
P-value: The probability of a z-statistic as extreme as the given one or more so (and in support of the alternative hypothesis) under the assumption that the null hypothesis is true.

So P-value =

(single) tail probability in 1-sided test case

twice the (single) tail probability in 2-sided test case.

$H_a : p > p_0$ P-value=tail probability



Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

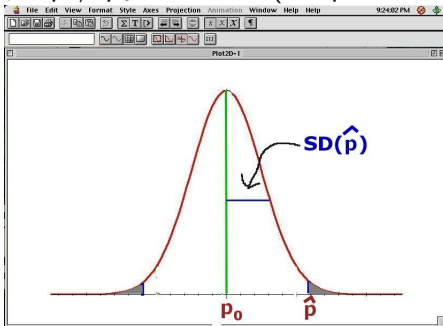
HT Vocabulary Details

P-value: The probability of a z-statistic as extreme as the given one or more so (and in support of the alternative hypothesis) under the assumption that the null hypothesis is true.

So P-value =

(single) tail probability in 1-sided test case
twice the (single) tail probability in 2-sided test case.

$$H_a : p \neq p_0 \text{ P-value} = 2(\text{tail probability})$$



Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

HT Vocabulary Details

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

P-value: The probability of a z-statistic as extreme as the given one or more so (and in support of the alternative hypothesis) under the assumption that the null hypothesis is true.

Significance Level α : The cutoff between "small" and "big" P-values.

HT Vocabulary Details

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

P-value: The probability of a z-statistic as extreme as the given one or more so (and in support of the alternative hypothesis) under the assumption that the null hypothesis is true.

Significance Level α : The cutoff between "small" and "big" P-values.

Small P-value means reject the null.

Big P-value means retain the null.

Logic of Hypothesis Testing

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

Procedure:

- 1 ...
- 2 ...
- 3 Consider the sampling distribution of \hat{p} which would hold if H_0 were true.
- 4 Retain H_0 if \hat{p} is reasonably consistent with this sampling distribution. Otherwise reject H_0 .
- 5 ...

Logic of Hypothesis Testing

Procedure:

- 1 ...
- 2 ...
- 3 Consider the sampling distribution of \hat{p} which would hold if H_0 were true.
- 4 Retain H_0 if \hat{p} is reasonably consistent with this sampling distribution. Otherwise reject H_0 .
- 5 ...

Three ways to carry out step 4:

- 1 Calculate a z-statistic and determine a p-value.
- 2 Calculate a z-statistic and compare with a critical value z^* .
- 3 Use a confidence interval.

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

Sample Size for a Given MOE

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

We wish to produce a 95% CI with a margin of error of .1%.
What sample size should we use?

Sample Size for a Given MOE

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

We wish to produce a 95% CI with a margin of error of .1%.
What sample size should we use?

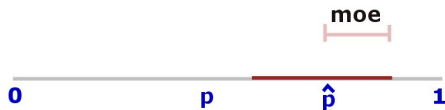
$$MOE = z^* \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

Sample Size for a Given MOE

We wish to produce a 95% CI with a margin of error of .1%.
What sample size should we use?

$$MOE = z^* \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

Margin of error picture.



Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

Sample Size for a Given MOE

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

We wish to produce a 95% CI with a margin of error of .1%.

What sample size should we use?

$$MOE = z^* \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

Algebra gives

$$n = \left(\frac{z^*}{MOE} \right)^2 \hat{p}\hat{q}$$

Sample Size for a Given MOE

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

We wish to produce a 95% CI with a margin of error of .1%.

What sample size should we use?

Algebra gives

$$n = \left(\frac{z^*}{MOE} \right)^2 \hat{p}\hat{q}$$

Here we have

$$n = \left(\frac{1.96}{.001} \right)^2 \hat{p}\hat{q} = 1960^2 \hat{p}\hat{q}.$$

Sample Size for a Given MOE

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

We wish to produce a 95% CI with a margin of error of .1%.

What sample size should we use?

Algebra gives

$$n = \left(\frac{z^*}{MOE} \right)^2 \hat{p}\hat{q}$$

Here we have

$$n = \left(\frac{1.96}{.001} \right)^2 \hat{p}\hat{q} = 1960^2 \hat{p}\hat{q}.$$

The challenge is we don't know \hat{p} until we conduct the sample.

Sample Size for a Given MOE

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

We wish to produce a 95% CI with a margin of error of .1%.

What sample size should we use?

Algebra gives

$$n = \left(\frac{z^*}{MOE} \right)^2 \hat{p}\hat{q}$$

Here we have

$$n = \left(\frac{1.96}{.001} \right)^2 \hat{p}\hat{q} = 1960^2 \hat{p}\hat{q}.$$

Three Approaches:

Sample Size for a Given MOE

We wish to produce a 95% CI with a margin of error of .1%.

What sample size should we use?

Algebra gives

$$n = \left(\frac{z^*}{MOE} \right)^2 \hat{p}\hat{q}$$

Here we have

$$n = \left(\frac{1.96}{.001} \right)^2 \hat{p}\hat{q} = 1960^2 \hat{p}\hat{q}.$$

Three Approaches:

1. **Conservative:** Just use $\hat{p} = .5$ since that gives the biggest possible value of $\hat{p}\hat{q}$.

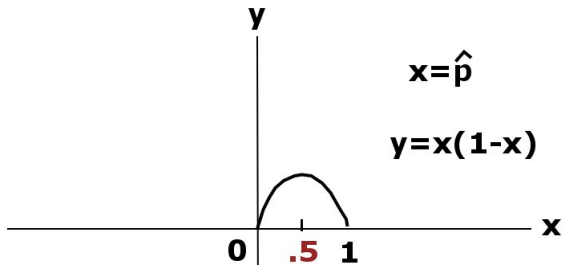
(Think about the graph of $y = x(1 - x)$.)

Sample Size for a Given MOE

We wish to produce a 95% CI with a margin of error of .1%.

What sample size should we use?

1. **Conservative:** Just use $\hat{p} = .5$ since that gives the biggest possible value of $\hat{p}\hat{q}$.



Sample Size for a Given MOE

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

We wish to produce a 95% CI with a margin of error of .1%.

What sample size should we use?

Here we have

$$n = \left(\frac{1.96}{.001} \right)^2 \hat{p}\hat{q} = 1960^2 \hat{p}\hat{q}.$$

Three Approaches:

1. **Conservative:** Just use $\hat{p} = .5$ since that gives the biggest possible value of $\hat{p}\hat{q}$.

Here we get

$$n = 1960^2 (.5)(.5) = 980^2 = 960,400$$

which explains why surveys DO NOT seek an MOE of .1%.

Sample Size for a Given MOE

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

Three Approaches:

1. **Conservative:** Just use $\hat{p} = .5$ since that gives the biggest possible value of $\hat{p}\hat{q}$.

Let's shift to producing a 95% CI with a margin of error of 2%.

What sample size should we use?

Sample Size for a Given MOE

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

Three Approaches:

1. **Conservative:** Just use $\hat{p} = .5$ since that gives the biggest possible value of $\hat{p}\hat{q}$.

Let's shift to producing a 95% CI with a margin of error of 2%.

What sample size should we use?

$$n = \left(\frac{1.96}{.02} \right)^2 \hat{p}\hat{q} = 98^2 \hat{p}\hat{q} = 9604 \hat{p}\hat{q}.$$

Sample Size for a Given MOE

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

Three Approaches:

1. **Conservative:** Just use $\hat{p} = .5$ since that gives the biggest possible value of $\hat{p}\hat{q}$.

Let's shift to producing a 95% CI with a margin of error of 2%.

What sample size should we use?

$$n = \left(\frac{1.96}{.02} \right)^2 \hat{p}\hat{q} = 98^2 \hat{p}\hat{q} = 9604 \hat{p}\hat{q}.$$

The conservative estimate here is $n = 9604 \cdot .25 = 2401$.

Sample Size for a Given MOE

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

Three Approaches:

Let's shift to producing a 95% CI with a margin of error of 2%.

What sample size should we use?

$$n = \left(\frac{1.96}{.02} \right)^2 \hat{p}\hat{q} = 98^2 \hat{p}\hat{q} = 9604 \hat{p}\hat{q}.$$

2. Use an estimate of \hat{p} :

Sample Size for a Given MOE

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

Three Approaches:

Let's shift to producing a 95% CI with a margin of error of 2%.

What sample size should we use?

$$n = \left(\frac{1.96}{.02} \right)^2 \hat{p}\hat{q} = 98^2 \hat{p}\hat{q} = 9604 \hat{p}\hat{q}.$$

2. Use an estimate of \hat{p} :

e.g. if we expect $\hat{p} \sim .2$, we might use this value leading to an estimate $n = 9604 \cdot .2 \cdot .8 = 1537$. But no guarantees.

Sample Size for a Given MOE

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

Three Approaches:

Let's shift to producing a 95% CI with a margin of error of 2%.

What sample size should we use?

$$n = \left(\frac{1.96}{.02} \right)^2 \hat{p}\hat{q} = 98^2 \hat{p}\hat{q} = 9604 \hat{p}\hat{q}.$$

3. If you expect \hat{p} to be in a range, use the most demanding \hat{p} within that range.

As long as your expectation is met, this is guaranteed to work.

Sample Size for a Given MOE

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

Three Approaches:

Let's shift to producing a 95% CI with a margin of error of 2%.

What sample size should we use?

$$n = \left(\frac{1.96}{.02} \right)^2 \hat{p}\hat{q} = 98^2 \hat{p}\hat{q} = 9604 \hat{p}\hat{q}.$$

3. If you expect \hat{p} to be in a range, use the most demanding \hat{p} within that range.

As long as your expectation is met, this is guaranteed to work.

For example if we expect $.05 \leq \hat{p} \leq .2$, use $\hat{p} = .2$ again leading to 1537.

Sample Size for a Given MOE

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

Three Approaches:

Let's shift to producing a 95% CI with a margin of error of 2%.

What sample size should we use?

$$n = \left(\frac{1.96}{.02} \right)^2 \hat{p}\hat{q} = 98^2 \hat{p}\hat{q} = 9604 \hat{p}\hat{q}.$$

3. If you expect \hat{p} to be in a range, use the most demanding \hat{p} within that range.

As long as your expectation is met, this is guaranteed to work.

How about if we expect $.2 \leq \hat{p} \leq .7$?

Sample Size for a Given MOE

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

Let's shift to producing a 95% CI with a margin of error of 2%.
What sample size should we use?

$$n = \left(\frac{1.96}{.02} \right)^2 \hat{p}\hat{q} = 98^2 \hat{p}\hat{q} = 9604 \hat{p}\hat{q}.$$

3. If you expect \hat{p} to be in a range, use the most demanding \hat{p} within that range.

As long as your expectation is met, this is guaranteed to work.

How about if we expect $.2 \leq \hat{p} \leq .7$?

The most demanding is $\hat{p} = .5$, so we use that and again arrive at 2401.

Sample Size for a Given MOE

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

Note the slow growth: A 100 fold increase in sample size reduces the MOE by only a factor of 10.

CPSC Example

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

Let p be the true proportion of smoke detectors homes in our city have.

CPSC Example

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

Let p be the true proportion of smoke detectors homes in our city have.

$$H_0 : p = .9, \quad H_A : p > .9$$

CPSC Example

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

Let p be the true proportion of smoke detectors homes in our city have.

$$H_0 : p = .9, \quad H_A : p > .9$$

$$SD(\hat{p}) = \sqrt{\frac{.9 \cdot .1}{400}} = .015.$$

CPSC Example

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

Let p be the true proportion of smoke detectors homes in our city have.

$$H_0 : p = .9, \quad H_A : p > .9$$

$$SD(\hat{p}) = \sqrt{\frac{.9 \cdot .1}{400}} = .015.$$

z-statistic

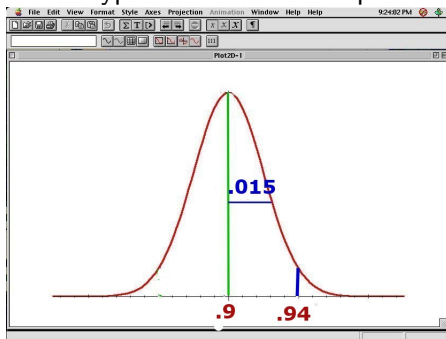
$$z = \frac{.94 - .9}{.015} = 2.67.$$

CPSC Example

z-statistic

$$z = \frac{.94 - .9}{.015} = 2.67.$$

z-statistic is the z-score of \hat{p} on the samp dist centered at the hypothesized value of p



Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

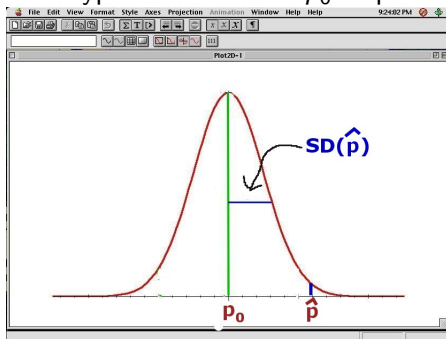
Proportion
Conditions

CPSC Example

z-statistic

$$z = \frac{.94 - .9}{.015} = 2.67.$$

z-statistic is the z-score of \hat{p} on the samp dist centered at the hypothesized value p_0 of p



Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

CPSC Example

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

z-statistic

$$z = \frac{.94 - .9}{.015} = 2.67.$$

Two primary ways to continue:

CPSC Example

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

z-statistic

$$z = \frac{.94 - .9}{.015} = 2.67.$$

Two primary ways to continue:

Method 1: Tail Prob = $P(\hat{p} > .94) = P(Z > 2.67) = .0038$

Since the test is 1-sided, P-value=tail probability=.0038.

CPSC Example

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

z-statistic

$$z = \frac{.94 - .9}{.015} = 2.67.$$

Two primary ways to continue:

Method 1: Tail Prob = $P(\hat{p} > .94) = P(Z > 2.67) = .0038$

Since the test is 1-sided, P-value=tail probability=.0038.

This is small, so conclusion is we reject H_0 .

CPSC Example

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

z-statistic

$$z = \frac{.94 - .9}{.015} = 2.67.$$

Two primary ways to continue:

Method 2: Say significance level is $\alpha = .05$.

CPSC Example

z-statistic

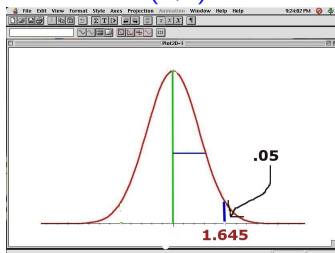
$$z = \frac{.94 - .9}{.015} = 2.67.$$

Two primary ways to continue:

Method 2: Say significance level is $\alpha = .05$.

Since this is a 1-sided test with $\alpha = .05$, the appropriate critical values is $z^* = 1.645$.

$N(0,1)$



Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

CPSC Example

z-statistic

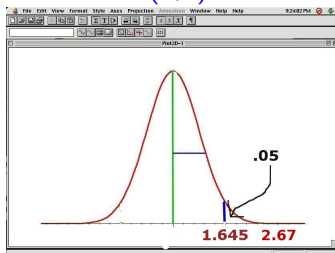
$$z = \frac{.94 - .9}{.015} = 2.67.$$

Two primary ways to continue:

Method 2: Say significance level is $\alpha = .05$.

Since our z-statistic of 2.67 is more extreme than $z^* = 1.645$ (and supports H_a), we reject H_0 at $\alpha = .05$.

$N(0,1)$



Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

CPSC Example

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

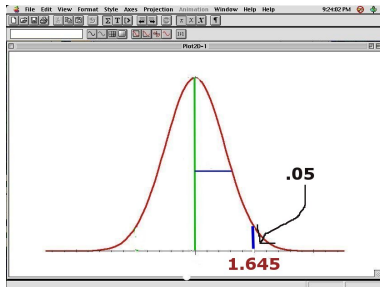
Power

Alternative
Hypotheses

Proportion
Conditions

Method 3: Using a CI. Still $\alpha = .05$.

The picture



shows that a 90% CI has the same critical value as an $\alpha = .05$ HT.

CPSC Example

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

Method 3: Using a CI. Still $\alpha = .05$.

$$SE(\hat{p}) = \sqrt{\frac{.94 \cdot .06}{400}} = .0119$$

CPSC Example

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

Method 3: Using a CI. Still $\alpha = .05$.

$$SE(\hat{p}) = \sqrt{\frac{.94 \cdot .06}{400}} = .0119$$

A 90% CI is

$$.94 \pm 1.645 \cdot .0119 = .94 \pm .0196 = (.9204, .9596)$$

CPSC Example

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

Method 3: Using a CI. Still $\alpha = .05$.

$$SE(\hat{p}) = \sqrt{\frac{.94 \cdot .06}{400}} = .0119$$

A 90% CI is

$$.94 \pm 1.645 \cdot .0119 = .94 \pm .0196 = (.9204, .9596)$$

Since all the values of p in our CI support H_a , the result of the HT is to reject the null.

Had even one been consistent with H_0 , we would have retained the null.

CPSC Example

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

Method 3: Using a CI. Still $\alpha = .05$.

$$SE(\hat{p}) = \sqrt{\frac{.94 \cdot .06}{400}} = .0119$$

A 90% CI is

$$.94 \pm 1.645 \cdot .0119 = .94 \pm .0196 = (.9204, .9596)$$

Because of the extra error associated with SE instead of SD, Method 3 is not arithmetically equivalent to Methods 1 and 2. Method 3 could reject the null while method 1 retained or vice-versa.

CPSC Example

Method 3: Using a CI. Still $\alpha = .05$.

$$SE(\hat{p}) = \sqrt{\frac{.94 \cdot .06}{400}} = .0119$$

A 90% CI is

$$.94 \pm 1.645 \cdot .0119 = .94 \pm .0196 = (.9204, .9596)$$

Because of the extra error associated with SE instead of SD, Method 3 is not arithmetically equivalent to Methods 1 and 2. Method 3 could reject the null while method 1 retained or vice-versa.

This small discrepancy is not of practical importance. It could be corrected by a more elaborate CI formula.

CPSC Example

Method 3: Using a CI. Still $\alpha = .05$.

$$SE(\hat{p}) = \sqrt{\frac{.94 \cdot .06}{400}} = .0119$$

A 90% CI is

$$.94 \pm 1.645 \cdot .0119 = .94 \pm .0196 = (.9204, .9596)$$

Because of the extra error associated with SE instead of SD, Method 3 is not arithmetically equivalent to Methods 1 and 2. Method 3 could reject the null while method 1 retained or vice-versa.

This small discrepancy is not of practical importance. It could be corrected by a more elaborate CI formula.

The general principle that many HT's can be done via a CI is an important fact.

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

CPSC Example

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

$$H_0 : p = .9, \quad H_A : p > .9$$

We saw that our 90% CI of (.9204, .9596) resulted in rejecting the null, since all values within the CI support H_A .

CPSC Example

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

$$H_0 : p = .9, \quad H_A : p > .9$$

We saw that our 90% CI of (.9204, .9596) resulted in rejecting the null, since all values within the CI support H_A .

Had the CI been e.g. (.89, .93) we would have retained H_0 since .9 is in the CI.

CPSC Example

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

$$H_0 : p = .9, \quad H_A : p > .9$$

We saw that our 90% CI of (.9204, .9596) resulted in rejecting the null, since all values within the CI support H_A .

Had the CI been e.g. (.89, .93) we would have retained H_0 since .9 is in the CI.

Had the CI been e.g. (.8, .84) we still would have retained H_0 since not all the values in the interval support H_A . (In fact none of them do.) This fits in with the idea that for logical purposes, our H_0 is really $p_a < .9$.

CPSC Example

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

$$H_0 : p = .9, \quad H_A : p > .9$$

We saw that our 90% CI of (.9204, .9596) resulted in rejecting the null, since all values within the CI support H_a .

Had the CI been e.g. (.89, .93) we would have retained H_0 since .9 is in the CI.

Had the CI been e.g. (.8, .84) we still would have retained H_0 since not all the values in the interval support H_a . (In fact none of them do.) This fits in with the idea that for logical purposes, our H_0 is really $p_a < .9$.

The 2-sided case is simpler. There, with H_0 being $p = p_0$ we retain H_0 exactly when p_0 is in the CI.

Power, Type I/II Errors, α , and β

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

Given $H_0 : p = p_0$, there are two ways an HT can report an inaccurate result:

Power, Type I/II Errors, α , and β

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

Given $H_0 : p = p_0$, there are two ways an HT can report an inaccurate result:

	H_0 true	H_0 false
Retain H_0	<i>Good</i>	<i>Type II Error</i> probability = β depends on value of p
Reject H_0	<i>Type I Error</i> probability = α	<i>Good</i> probability = $1-\beta$ = <i>power</i>

Power, Type I/II Errors, α , and β

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

Given $H_0 : p = p_0$, there are two ways an HT can report an inaccurate result:

Type I Error Examples:

- a: False Positive in a diagnosis; i.e. deciding a person is sick when they really are not. (H_0 : *The person is well.*)
- b: Convicting an innocent person. (H_0 : *The person is innocent.*)
- c: Producer Risk; the chance that a good good shipment erroneously fails a a test for quality. (H_0 : *A product meets a specification.*)

Power, Type I/II Errors, α , and β

Given $H_0 : p = p_0$, there are two ways an HT can report an inaccurate result:

Type I Error Examples:

- a: False Positive in a diagnosis; i.e. deciding a person is sick when they really are not. (H_0 : *The person is well.*)
- b: Convicting an innocent person. (H_0 : *The person is innocent.*)
- c: Producer Risk; the chance that a good good shipment erroneously fails a a test for quality. (H_0 : *A product meets a specification.*)

Type II Error Examples:

- a: False Negative; missing a sick person.
- b: Letting a guilty person go free.
- c: Consumer Risk; the chance that a bad shipment erroneously passes a a test for quality.

Three Kinds of H'_a s

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

H_a for a prop. HT can be any of the three possibilities

Two-Sided $p \neq p_0$

One-Sided $p > p_0$

One-Sided $p < p_0$

Three Kinds of H'_a s

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

H_a for a prop. HT can be any of the three possibilities

Two-Sided $p \neq p_0$

One-Sided $p > p_0$

One-Sided $p < p_0$

One chooses among these based on the question being studied.

Three Kinds of H'_a s

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

H_a for a prop. HT can be any of the three possibilities

Two-Sided $p \neq p_0$

One-Sided $p > p_0$

One-Sided $p < p_0$

One chooses among these based on the question being studied.

A question like “Is there strong evidence that p has *changed* ...” would point to 2-sided.

Three Kinds of H'_a s

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

H_a for a prop. HT can be any of the three possibilities

Two-Sided $p \neq p_0$

One-Sided $p > p_0$

One-Sided $p < p_0$

One chooses among these based on the question being studied.

A question like “Is there strong evidence that p has *increased* ...” would point to 1-sided.

Three Kinds of H'_a s

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

H_a for a prop. HT can be any of the three possibilities

Two-Sided $p \neq p_0$

One-Sided $p > p_0$

One-Sided $p < p_0$

One chooses among these based on the question being studied.

The value of \hat{p} never plays a role in formulating hypotheses!

Three Kinds of H'_a s

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

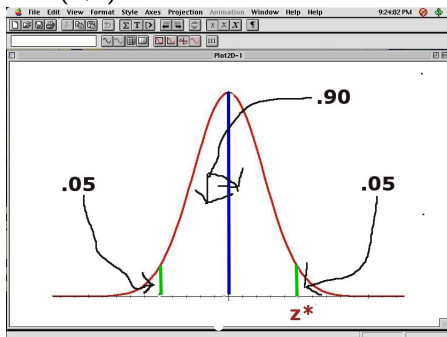
Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

$N(0,1)$ and the critical value z^*



Three Kinds of H'_a s

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

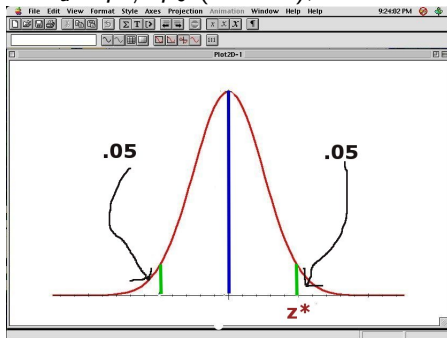
Power

Alternative
Hypotheses

Proportion
Conditions

$N(0,1)$

$H_a : p \neq p_0$ (2-sided), $\alpha = .10$



Three Kinds of H'_a 's

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

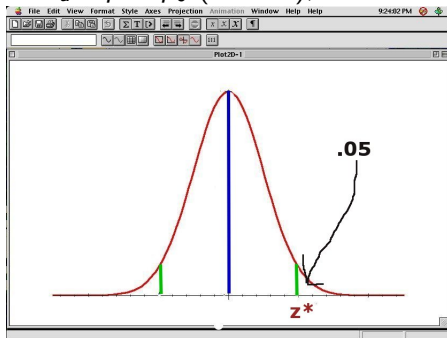
Power

Alternative
Hypotheses

Proportion
Conditions

$N(0,1)$

$H_a : p > p_0$ (1-sided), $\alpha = .05$



Three Kinds of H'_a 's

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

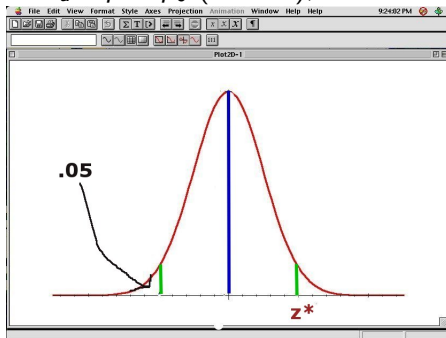
Power

Alternative
Hypotheses

Proportion
Conditions

$N(0,1)$

$H_a : p < p_0$ (1-sided), $\alpha = .05$



Conditions for Prop Tests

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

- plausible independence
- random sampling
- 10% condition
- success/failure

What Happens if Not Satisfied:

Conditions for Prop Tests

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

- plausible independence
- random sampling
- 10% condition
- success/failure

What Happens if Not Satisfied:

random sampling -
could be critical;
might be ok if "representative"
representative hard/impossible to define

Conditions for Prop Tests

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

- plausible independence
- random sampling
- 10% condition
- success/failure

What Happens if Not Satisfied:

plausible independence -
could be critical
sometimes just a working hypothesis

Conditions for Prop Tests

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

- plausible independence
- random sampling
- 10% condition
- success/failure

What Happens if Not Satisfied:

10% condition -

results in overestimation of samp. dist. st dev
gradual breakdown in formulas, not method

Conditions for Prop Tests

Math 1710
Class 28

V2

Last Time

Sample Size
for a Given
MOE

Three ways to
Carry Out an
HT

Power

Alternative
Hypotheses

Proportion
Conditions

- plausible independence
- random sampling
- 10% condition
- success/failure

What Happens if Not Satisfied:

success/failure -

progressive reduction of accuracy

accuracy varies regardless for smaller values of n