Math 1710 Class 31

t - Inference for Means

Dr. Back

Nov. 9, 2009

Chicken Contamination

Suppose:

1. 33% of 75 Perdue chickens contaminated.
2. 45% of 75 Store Brand chickens contaminated.
3. 56% of 75 Tyson chickens contaminated.

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Questions:

1. Purdue safer than Store Brand?
2. Tyson safer than Store Brand?
3. Tyson different in safety than Store Brand?
4. Confidence interval for difference in safety between Store Brand and Tyson?

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Notation: Let p_1 denote the proportion of Purdue which are contaminated and p_2 the proportion for Store Brand.

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Hypotheses:

- H_0: $p_1 = p_2$ $(or\ p_1 \geq p_2)$
- H_a: $p_1 < p_2$

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$$\hat{p}_{pooled} = \frac{.33 \cdot 75 + .45 \cdot 75}{75 + 75} = .39$$

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$$\hat{p}_{pooled} = \frac{.33 \cdot 75 + .45 \cdot 75}{75 + 75} = .39$$

$$SE_{pooled}(\hat{p}_1 - \hat{p}_2) = \sqrt{.39 \cdot .61 \left(\frac{1}{75} + \frac{1}{75} \right)} = .0796.$$

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$$z = \frac{\hat{p}_1 - \hat{p}_2}{SE_{pooled}} = \frac{-.12}{.0796} = -1.51.$$

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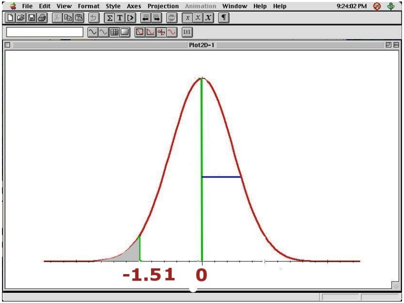
N(0,1) 

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$$z = \frac{\hat{p}_1 - \hat{p}_2}{SE_{pooled}} = \frac{-.12}{.0796} = -1.51.$$

P-value = tail probability = $P(Z < -1.51) = .0655$.
At a level of $\alpha = .05$, we'd retain H_0.
Purdue might not be safer.

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Notation: Let p_2 denote the proportion of Store Brand which are contaminated and p_3 the proportion for Tyson.

Hypotheses:

- H_0: $p_3 = p_2$ $(or\ p_3 \geq p_2)$
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$$\hat{p}_{pooled} = \frac{.45 \cdot 75 + .56 \cdot 75}{75 + 75} = .505$$

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$$\hat{p}_{pooled} = \frac{.45 \cdot 75 + .56 \cdot 75}{75 + 75} = .505$$

$$SE_{pooled}(\hat{p}_2 - \hat{p}_3) = \sqrt{.505 \cdot .495 \left(\frac{1}{75} + \frac{1}{75}\right)} = .0816.$$

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$$z = \frac{\hat{p}_2 - \hat{p}_3}{SE_{pooled}} = \frac{-.11}{.0816} = -1.35.$$

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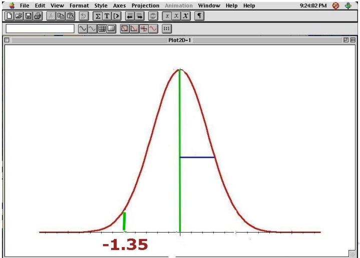
$$z = \frac{\hat{p}_2 - \hat{p}_3}{SE_{pooled}} = \frac{-.11}{.0816} = -1.35.$$

Which side provides as much or more support for H_a of $p_3 < p_2$?



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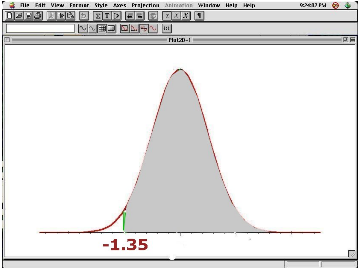
Which side provides as much or more support for $p_3 < p_2$?



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Our statistic provides no support for H_a so we immediately retain H_0.

It is a matter of convention whether we'd view the p-value as .5 or even larger.

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Notation: Let p_2 denote the proportion of Store Brand which are contaminated and p_3 the proportion for Yson.

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Still $\hat{p}_{pooled} = .505$, $SE_{pooled}(\hat{p}_2 - \hat{p}_3) = .0816$, $z = -1.35$.

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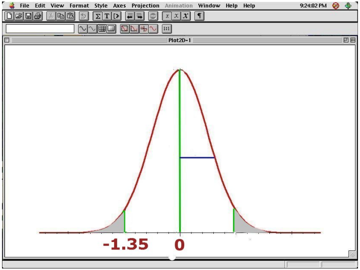

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tail probability $= P(Z < -1.35) = .0885$.
P-value = 2(tail probability)=2(.0885)=.177
At a level of $\alpha = .05$, we'd retain H_0.
Tyson might not have a different level of safety than Store Brand.

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Confidence interval for difference in safety between Store Brand and Tyson?

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Confidence interval for difference in safety between Store Brand and Tyson?

$$SE_{pooled} = \sqrt{\frac{.45 \cdot .55}{75} + \frac{.56 \cdot .44}{75 + 75}} = .0812$$

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Confidence interval for difference in safety between Store Brand and Tyson?

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A 95% CI for $p_2 - p_3$ would be

$$-.11 \pm 1.96 \cdot .0812 = -.11 \pm .159 = (-.269, .049)$$

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The fact that this CI contains 0 is another way of doing the last 2 HT's.

Coffee Machine

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9.9	9.6	9.8	9.8
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Is there evidence that the machine is shortchanging customers?

Coffee Machine

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A natural HT situation.

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We'll summarize the data by its mean $\bar{x} = 9.845$ and its standard deviation $s = .1986$.

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Notation: Let μ denote the mean amount of coffee in a dispensed cup.

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Notation: Let μ denote the mean amount of coffee in a dispensed cup.

Hypotheses:

- H_0: $\mu = 10$ *(or* $\mu \geq 10$*)*
- H_a: $\mu < 10$

Coffee Machine

Is there evidence that the machine is shortchanging customers?

Recall by the CLT that the sampling distribution of $\bar{x}$ is

$$N(\mu, \frac{\sigma}{\sqrt{n}})$$

when n is large.

Coffee Machine

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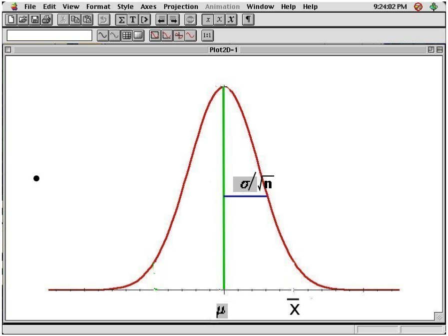
$$N(\mu, \frac{\sigma}{\sqrt{n}})$$



Coffee Machine

Is there evidence that the machine is shortchanging customers?

As usual with HT's, we are interested in whether the observed statistic of $\bar{x} = 9.845$ is reasonably consistent with the sampling distribution assuming H_0 is true.

Coffee Machine

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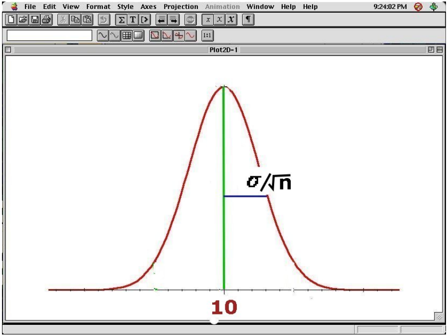
$$N(10, \frac{\sigma}{\sqrt{n}})$$



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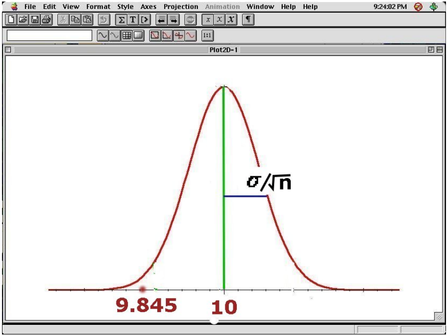
Is there evidence that the machine is shortchanging customers?

$N(10, \frac{\sigma}{\sqrt{n}})$ with $\bar{x}$



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What is the obvious approximation?

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Answer: Use $s = .1986$ instead of σ.

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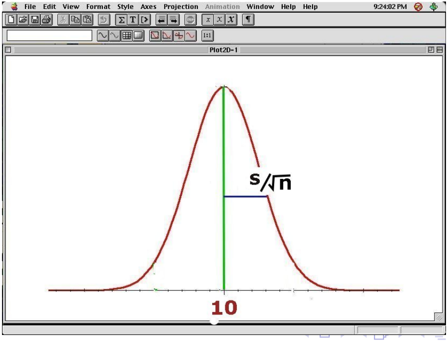
Answer: Use $s = .1986$ instead of σ.

$$N(10, \frac{s}{\sqrt{n}})$$



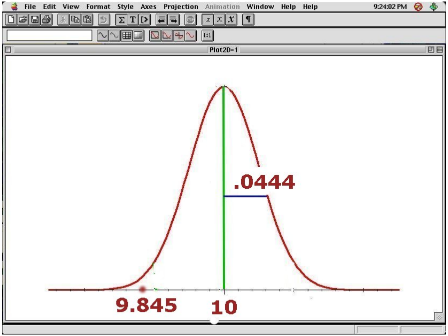
Coffee Machine

Is there evidence that the machine is shortchanging customers?

Since the standard error is

$$SE(\bar{x}) = \frac{s}{\sqrt{n}} = \frac{.1986}{\sqrt{20}} = .0444$$

$$N(10, .0444) \text{ with } \bar{x} = 9.845.$$



Coffee Machine

Is there evidence that the machine is shortchanging customers?

If $s = \sigma$, we'd look at a Z-statistic

$$Z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{9.845 - 10}{\frac{.1986}{\sqrt{20}}}$$

where we've written H_0 more abstractly as $\mu = \mu_0$, μ_0 being the hypothesized value, 10 in this case.

Coffee Machine

Is there evidence that the machine is shortchanging customers?

Because s will not exactly match σ, we actually get a bit of extra error here. This is compensated for by viewing

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{9.845 - 10}{\frac{.1986}{\sqrt{20}}} = \frac{-.155}{.0444} = -3.49.$$

as a *t-Statistic*.

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$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{9.845 - 10}{\frac{.1986}{\sqrt{20}}} = \frac{-.155}{.0444} = -3.49.$$

as a *t-Statistic*.

Since the error in approximating σ by s varies with the sample size, there is a different t-distribution for each sample size.

These are labeled by the "degrees of freedom" which for a 1-sample t-test is:

$$df = n - 1.$$

Coffee Machine

Is there evidence that the machine is shortchanging customers?

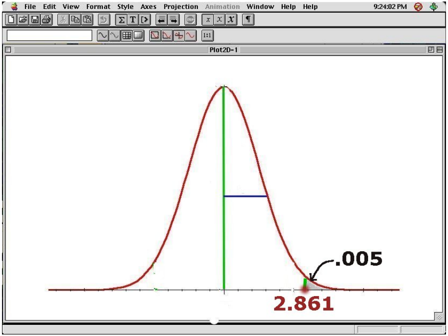
One tail probability		0.10	0.05	0.025	0.01	0.005
Table T Values of t_α	df					
	1	3.078	6.314	12.706	31.821	63.657
	2	1.886	2.920	4.303	6.965	9.925
	3	1.638	2.353	3.182	4.541	5.841
	4	1.533	2.132	2.776	3.747	4.604
	17	1.333	1.740	2.110	2.567	2.898
	18	1.330	1.734	2.101	2.552	2.878
	19	1.328	1.729	2.093	2.539	2.861
	20	1.325	1.725	2.086	2.528	2.845
	21	1.323	1.721	2.080	2.518	2.831

These are all critical values t^*.
For example $P(t > 1.328) = .10$ for the t distribution with 19 df.

Coffee Machine

Is there evidence that the machine is shortchanging customers?

Our t-statistic of -3.49 is more extreme than any on the df=19 row of the table.
The picture



shows what the critical value $t^* = 2.861$ for a tail prob. of .005

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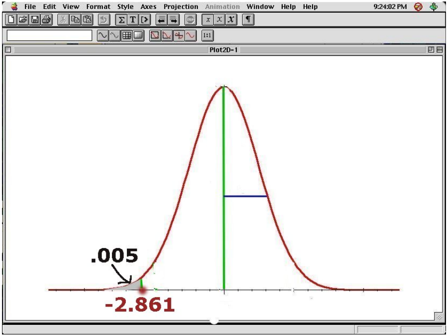
So by symmetry

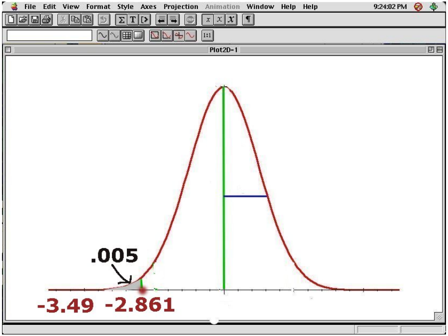


$P(T < -2.861) = .005$ as well.

Coffee Machine

Is there evidence that the machine is shortchanging customers?

So our tail probability and p-value are both less than .005 and we reject the null. The machine does appear to be shortchanging.



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Sps. our t-statistic had been 2.00 with the same 1-sided hypotheses

- H_0: $\mu = 10$ *(or* $\mu \geq 10$*)*
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What P-value would we report?

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Sps. our t-statistic had been 2.00 with the same 1-sided hypotheses

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What P-value would we report?

One tail probability		0.10	0.05	0.025	0.01	0.005
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Answer: A tail probability and P-value of between .025 and .05.

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Sps. instead our t-statistic had been 2.00 with 2-sided hypotheses

- H_0: $\mu = 10$
- H_a: $\mu \neq 10$

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One tail probability	df	0.10	0.05	0.025	0.01	0.005
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Answer: Our tail probability is still between .025 and .05 but our P-value is now between .05 and .10.