Math 1710 Class 25

Sampling, Confidence Intervals

Dr. Back

Oct. 26, 2009

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Our SRS is units 16 17 20 29 and 05.

Average Area?

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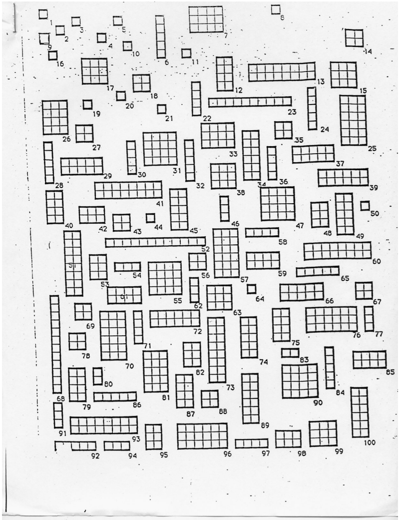

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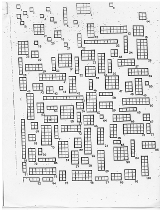
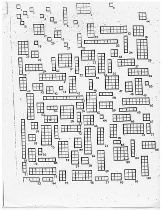


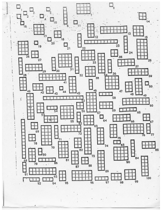
Judgement Sample: Each person picks 5 representative rectangles, and computes their average area.

Average Area?



SRS: Each person uses a table of random numbers to choose 5 random rectangles, and computes their average area.

Average Area?



Stratified Random Sample: Call a rectangle big if its area is above 5; small otherwise.

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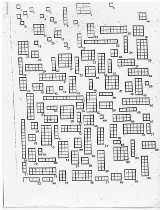

Stratified Random Sample: Call a rectangle big if its area is above 5; small otherwise.

Have 70% of people find averages of big rectangles and 30% study small ones.

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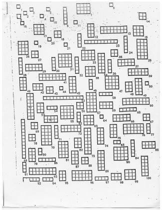

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The numbers .48 and .52 come from the fact that there are 48 small and 52 big rectangles.

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Have 70% of people find averages of big rectangles and 30% study small ones.

The fractions 70% and 30% come from the following calculation:

Type	n_i	σ_i	$n_i\sigma_i$
small rectangles	48	1.557	74.74
big rectangles	52	3.847	200.04

$$\frac{200.04}{200.04 + 74.74} = .73 \sim 70\%$$

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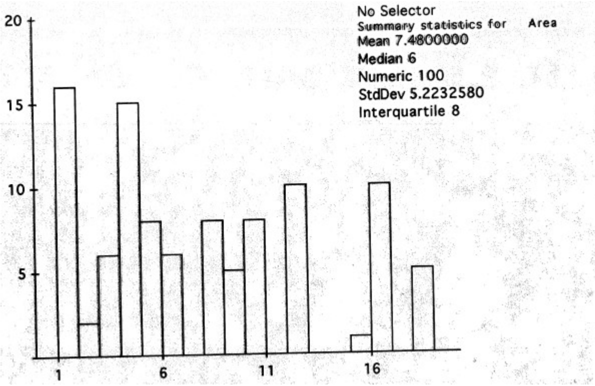
These proportions give the stratified random sample with smallest variance for a given sample size.

Average Area?

Histogram of Actual Areas



Average Area?

Frequency Breakdown of Actual Areas

Area

Frequency breakdown of Area
No Selector

Group	Count	Cumulative Count	%	Cumulative %
1	16	16	16	16
2	2	18	2	18
3	6	24	6	24
4	15	39	15.0	39
5	8	47	8	47
6	6	53	6	53.0
8	8	61	8	61
9	5	66	5	66
10	8	74	8	74
12	10	84	10	84
15	1	85	1	85
16	10	95	10	95
18	5	100	5	100
total	100			

Newspaper Poll

Poll size $n = 400$.
144 say yes, so

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Recall $SD(\hat{p}) = \sqrt{\frac{pq}{n}}$, $SE(\hat{p}) = \sqrt{\frac{\hat{p}\hat{q}}{n}}$

Newspaper Poll

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The sampling distribution of $\hat{p}$ is $N(p, SD(\hat{p}))$.

p	**SD($\hat{p}$)**
.36	.0240
.35	.0238
.3	.0229
.4	.0245
.45	.0249
.5	.0250
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Our sampling distribution of $\hat{p}$ is $N(p, .024)$ approximately. For which p is $\hat{p} = .36$ reasonably consistent with this?

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Use the sampling distribution of $\hat{p}$ ($N(p, .24)$ in our example) (together with the confidence level) to decide which p are reasonably consistent.

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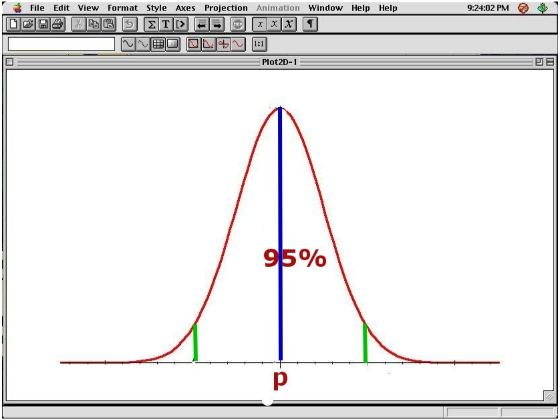

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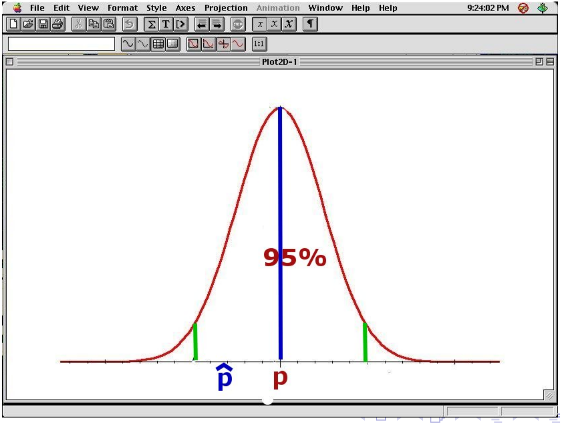

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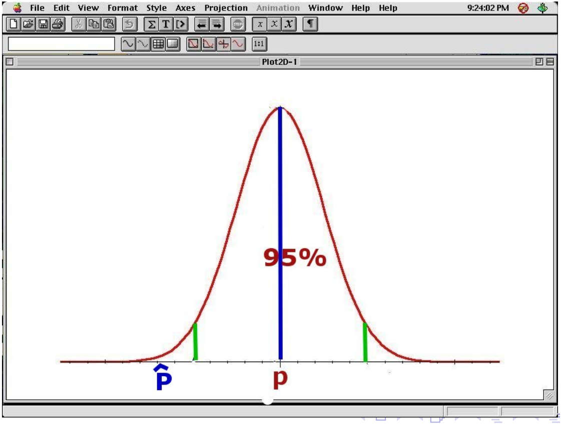

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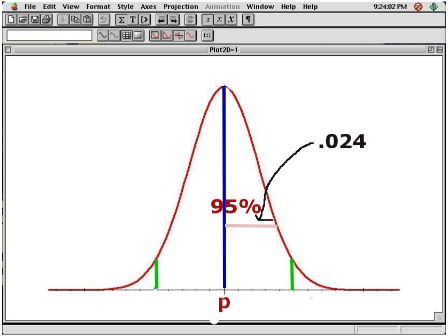
At the level of the 68-95-99.7 rule, middle 95% means within 2 standard deviations.



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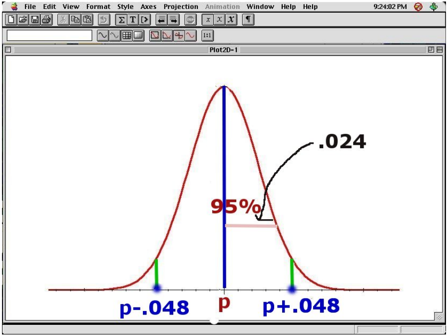

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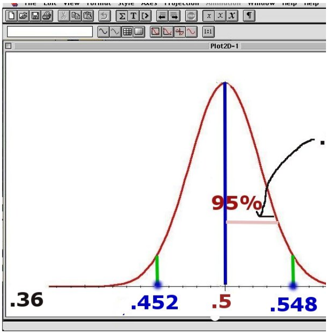
This shows $p = .5$ should not be in the CI:



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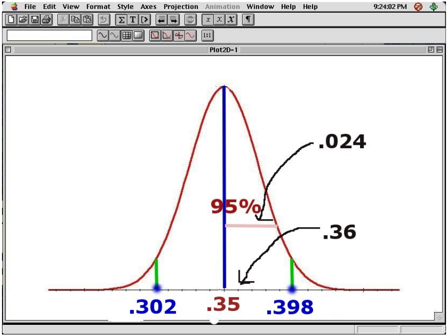
This shows $p = .35$ should be in the CI:



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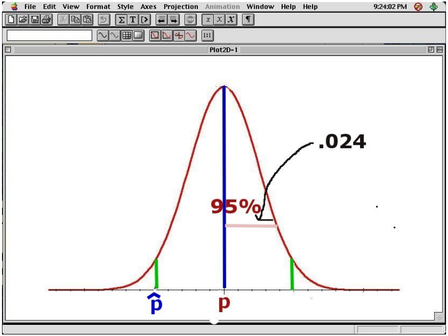

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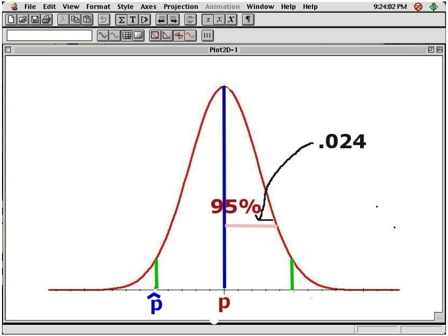

So the upper limit is $\hat{p} + .048 = .408$.

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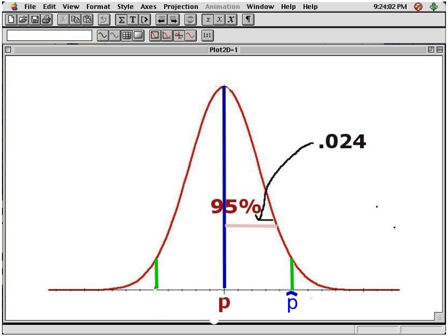

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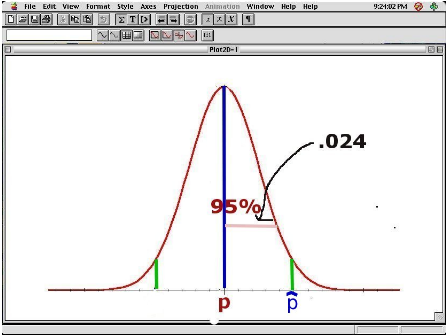

So the lower limit is $\hat{p} - .048 = .312$.

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Here

$$\hat{p} \pm z^* SE(\hat{p}) = .36 \pm (1.96)(.024) = .36 \pm .047 = (.313, .407)$$

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We are 95% confident that the true value of p is between .313 and .407.