Math 1710 Class 8

Normal Distributions
Dr. Back

Sep. 14, 2009

A Continuous Distribution

There's a normal distribution with any mean μ or $\sigma > 0$.

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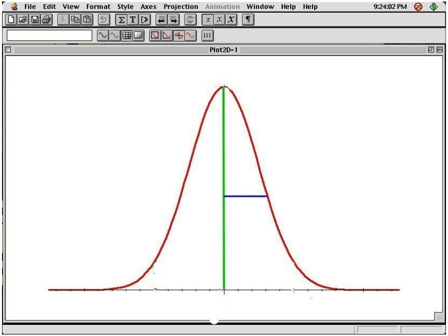
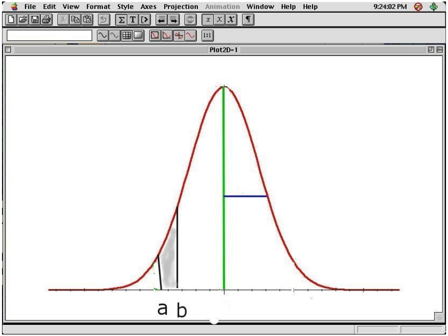
$N(\mu, \sigma)$



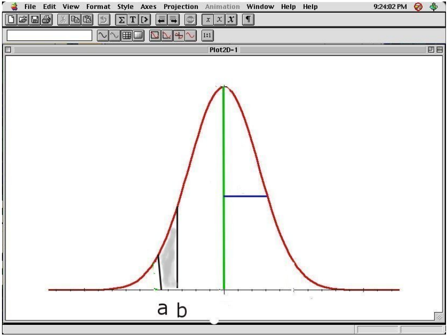
A Continuous Distribution

$N(\mu, \sigma)$



A Continuous Distribution

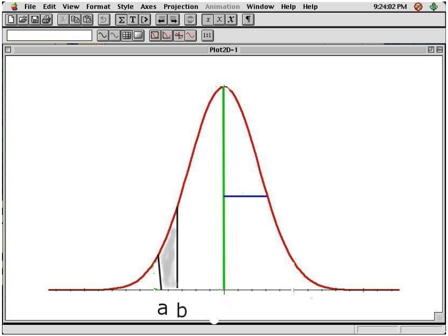
$$N(\mu, \sigma)$$



Area corresponds to probability.

A Continuous Distribution

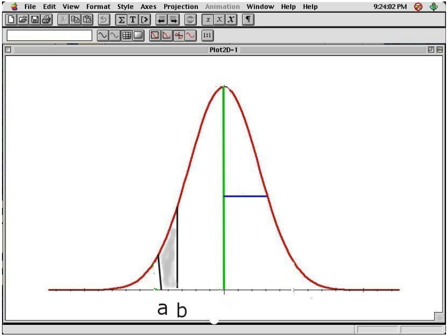
$$N(\mu, \sigma)$$



The entire area under the curve is 1.

A Continuous Distribution

$$N(\mu, \sigma)$$



The area between *a* and *b* is the probability of a value *x* falling within that range.

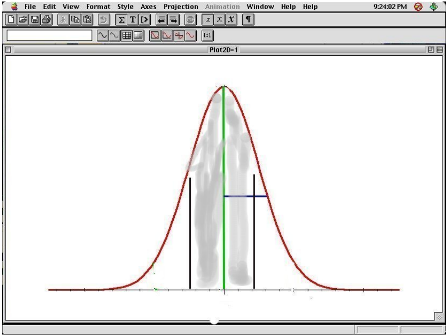
The 68 – 95 – 99.7 Rule

68% within 1 standard deviation of the mean.

The 68 − 95 − 99.7 Rule

68% within 1 standard deviation of the mean.

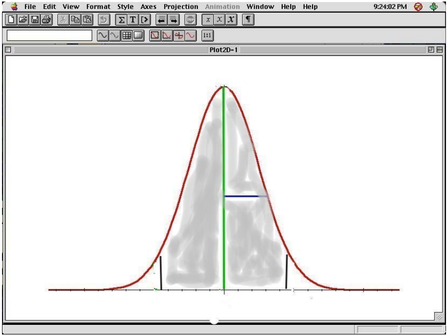
$N(\mu, \sigma)$



The 68 — 95 — 99.7 Rule

95% within 2 standard deviations of the mean.

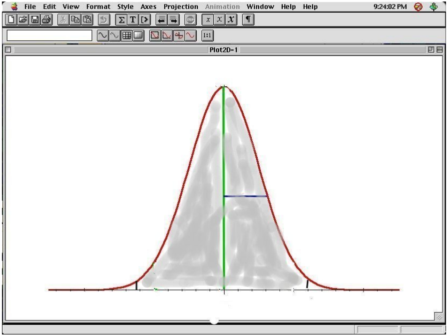
$N(\mu, \sigma)$



The 68 – 95 – 99.7 Rule

99.7% within 3 standard deviations of the mean.

$N(\mu, \sigma)$



The Standard Normal Distribution

Standard normal is the case $\mu = 0$ and $\sigma = 1$.

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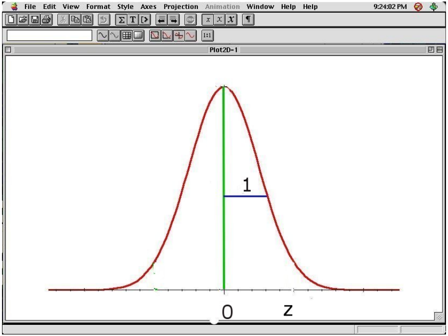
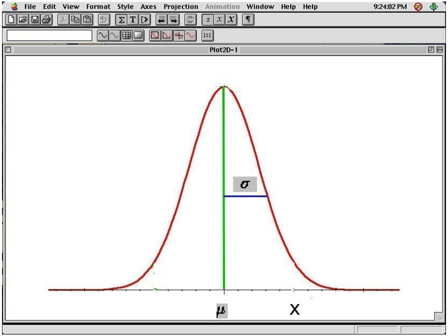
N(0,1)



The Standard Normal Distribution

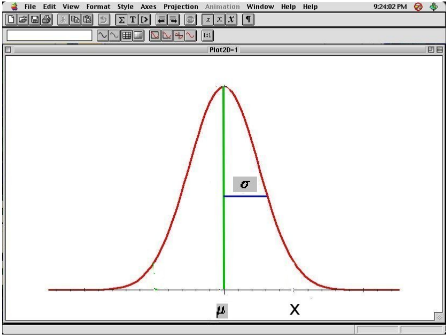
A general normal distribution:

$$N(\mu, \sigma)$$



The Standard Normal Distribution

$N(\mu, \sigma)$

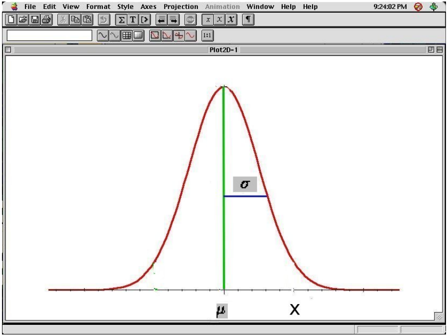


Can convert from a general $N(\mu, \sigma)$ to $N(0, 1)$ via the Z-score.

$$z = \frac{x - \mu}{\sigma}$$

The Standard Normal Distribution

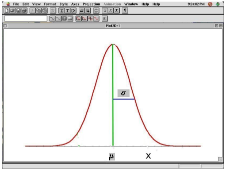
$$N(\mu, \sigma)$$



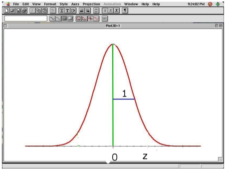
$$z = \frac{x - \mu}{\sigma}$$

The Z score is just the offset from the mean in standard deviation units.

The Standard Normal Distribution



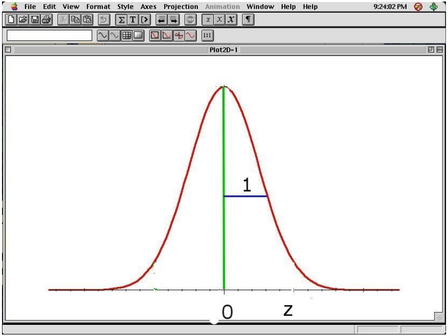
$$z = \frac{x-\mu}{\sigma}$$



This transformation preserves area and probability.

Using Table Z

N(0,1)



Using Table Z

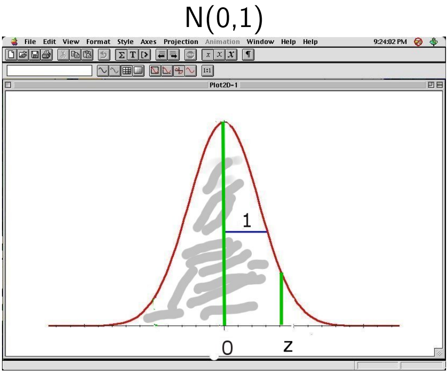


Table Z gives us the area to the left on the standard normal.

Using Table Z

N(0,1)

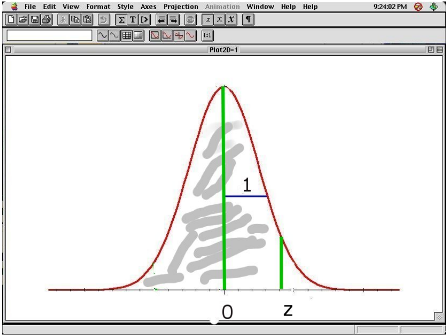


Table Z gives us the area to the left on the standard normal.
i.e. $P(Z < z)$

Using Table Z

Table Z gives us the area to the left on the standard normal.
For example $P(Z < 1) = .8413$ since

	Second decimal place in z									
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015

UsingTable Z

Table Z gives us the area to the left on the standard normal. For example $P(Z < 1) = .8413$ since

	Second decimal place in z									
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1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015

And $P(Z < 1.16) = .8770$.

UsingTable Z

Table Z gives us the area to the left on the standard normal.
Note you use row 1.1 and column .06 for this!

Using Table Z

We can also get the area under the curve between two values.

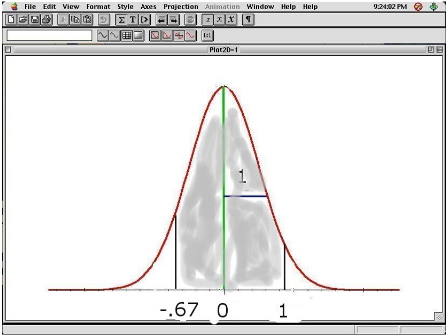
Using Table Z

For example $P(-.67 < Z < 1) =?$

Using Table Z

For example $P(-.67 < Z < 1) = ?$

N(0,1)

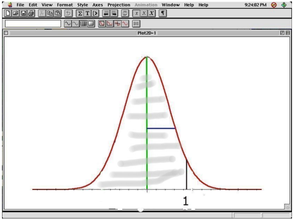


Using Table Z

Table Z tells us $P(-.67 < Z < 1) = .8413 - .2514 = .5899$.

Using Table Z

The reason for the subtraction
$P(-.67 < Z < 1) = P(Z < 1) - P(Z < -.67)$ is:



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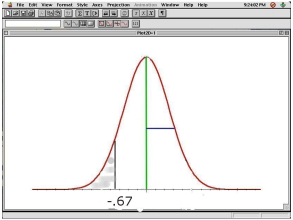

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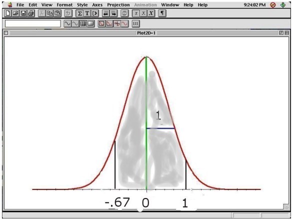
$=$ 

Mileage Example

Suppose a normal model N(24 mpg, 6 mpg) describes fuel efficiency of cars in a region:

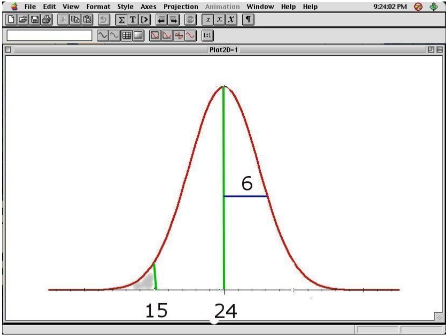
Mileage Example

Percent of cars with mileage below 15?

Mileage Example

Percent of cars with mileage below 15?

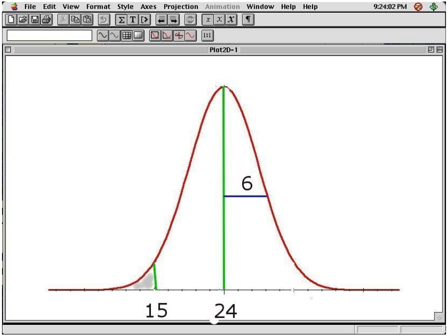
N(24,6)



Mileage Example

Percent of cars with mileage below 15?

N(24,6)

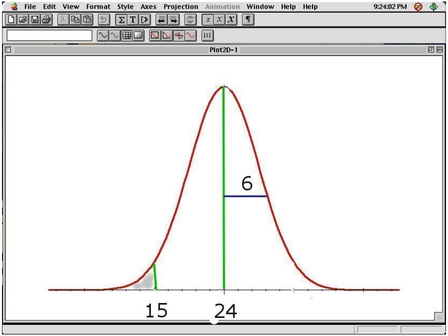


We understand a general $N(\mu, \sigma)$ by reducing to $N(0,1)$.

Mileage Example

Percent of cars with mileage below 15?

N(24,6)



We understand a general $N(\mu, \sigma)$ by reducing to $N(0, 1)$.
The Z score of 15 is ?

Mileage Example

Percent of cars with mileage below 15?

The Z score of 15 is ?

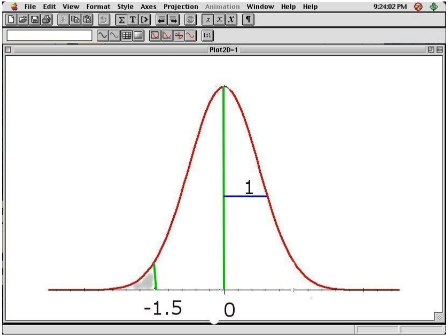
$$z = \frac{15 - 24}{6} = -1.5$$

Mileage Example

Percent of cars with mileage below 15?

$$z = \frac{15 - 24}{6} = -1.5$$

N(0,1)



Mileage Example

Percent of cars with mileage below 15?

N(0,1)

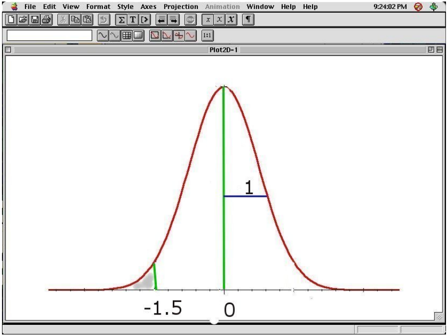


Table Z tells us $P(Z < -1.5) = .0668$.
6.7% of cars will have mileage below 15 mpg.

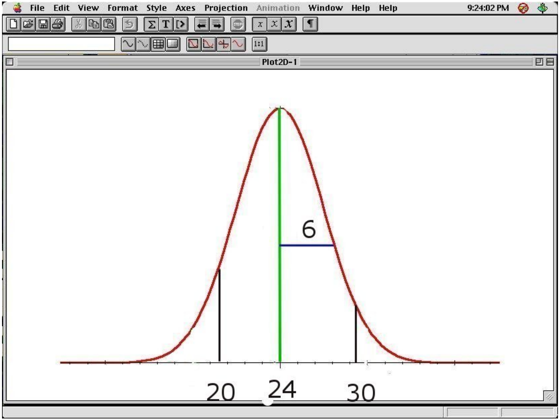
Mileage Example

% between 20 and 30?

Mileage Example

% between 20 and 30?

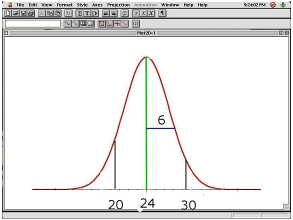
N(24,6)



Mileage Example

% between 20 and 30?

N(24,6)

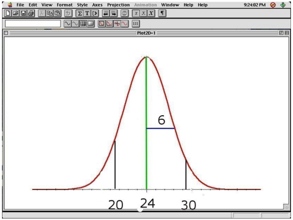


The Z score of 20 is ?

Mileage Example

% between 20 and 30?

N(24,6)

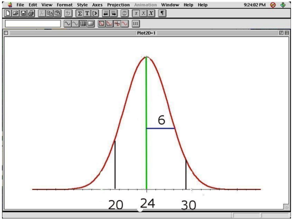


$$z_1 = \frac{20 - 24}{6} = -.67$$

Mileage Example

% between 20 and 30?

N(24,6)

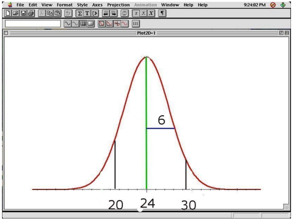


The Z score of 20 is $z_1 = -.67$?

Mileage Example

% between 20 and 30?

N(24,6)

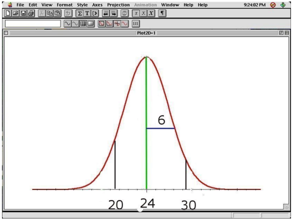


The Z score of 20 is $z_1 = -.67$?
The Z score of 30 is ?

Mileage Example

% between 20 and 30?

N(24,6)



The Z score of 20 is $z_1 = -.67$?
The Z score of 30 is ?

$$z_2 = \frac{30 - 24}{6} = 1$$

Mileage Example

Math 1710
Class 8

V4u

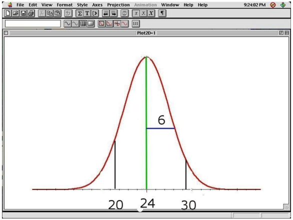
Last Time

Normal Distributions

Working With Normal Distributions

% between 20 and 30?

N(24,6)



The Z score of 20 is $z_1 = -.67$?
The Z score of 30 is $z_2 = 1$.

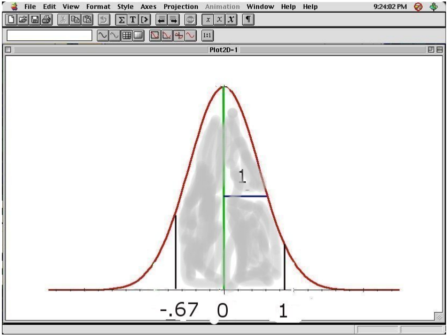
Mileage Example

% between 20 and 30?

The Z score of 20 is $z_1 = -.67$?

The Z score of 30 is $z_2 = 1$.

N(0,1)



Mileage Example

% between 20 and 30?

The Z score of 20 is $z_1 = -.67$?

The Z score of 30 is $z_2 = 1$.

N(0,1)

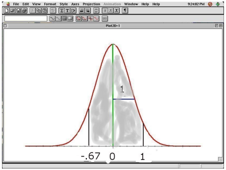
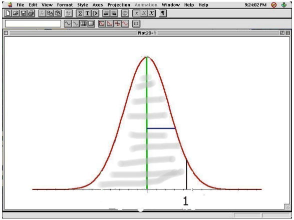


Table Z tells us $P(-.67 < Z < 1) = .8413 - .2514 = .5899$.

58.7% of cars will have mileage between 20 and 30 mpg.

Mileage Example

The reason for the subtraction
$P(-.67 < Z < 1) = P(Z < 1) - P(Z < -.67)$ is:



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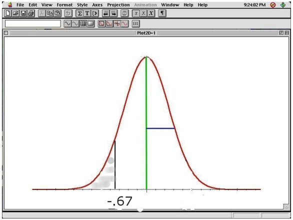

Mileage Example

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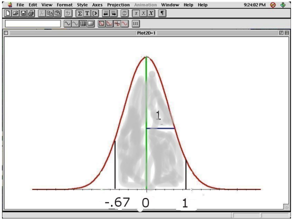
$=$ 

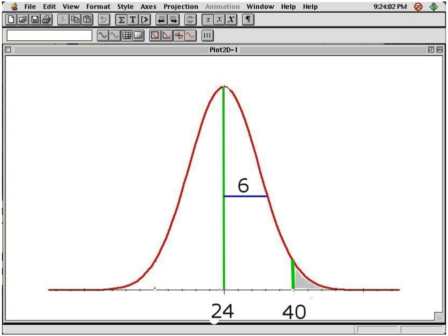
Mileage Example

% of cars above 40?

Mileage Example

% of cars above 40?

N(24,6)

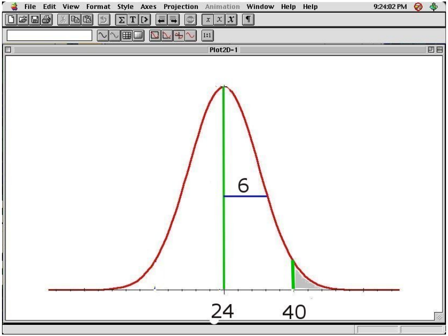


The Z score of 40 is ?

Mileage Example

% of cars above 40?

N(24,6)



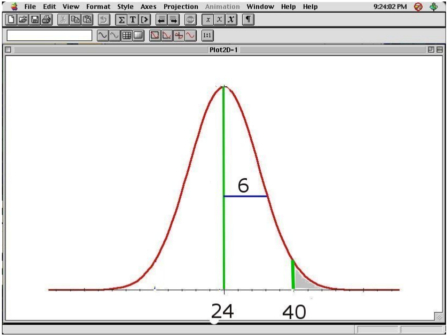
The Z score of 40 is ?

$$z = \frac{40 - 24}{6} = 2.67$$

Mileage Example

% of cars above 40?

N(24,6)



$$z = \frac{40 - 24}{6} = 2.67$$

Mileage Example

% of cars above 40?

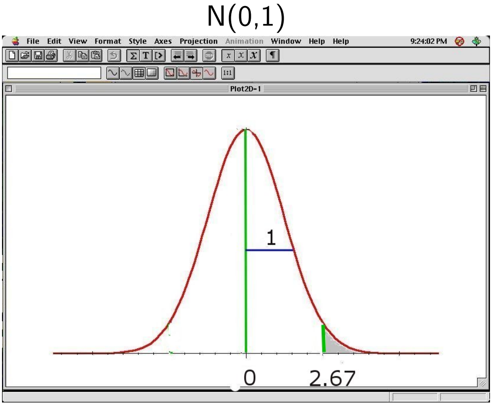


Table Z tells us $P(Z < 2.67) = .9962$.
Therefore $P(Z > -2.67) = 1 - .9962 = .0038$.
0.38% of cars will have mileage above 40 mpg.

Mileage Example

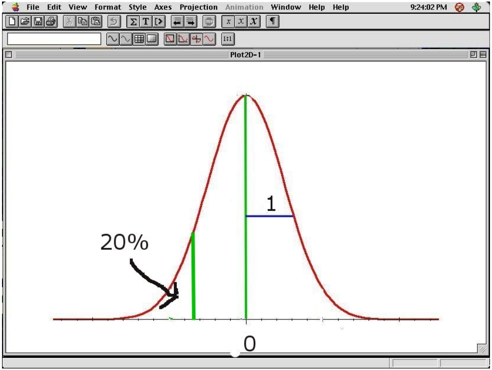
Worst 20% of cars?

Mileage Example

Worst 20% of cars?

Now we need to start with the $N(0, 1)$ picture.

N(0,1)



Mileage Example

Worst 20% of cars?

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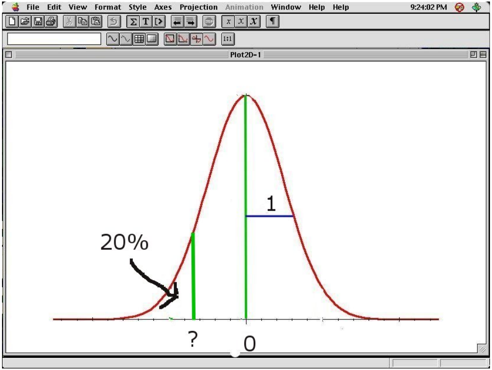
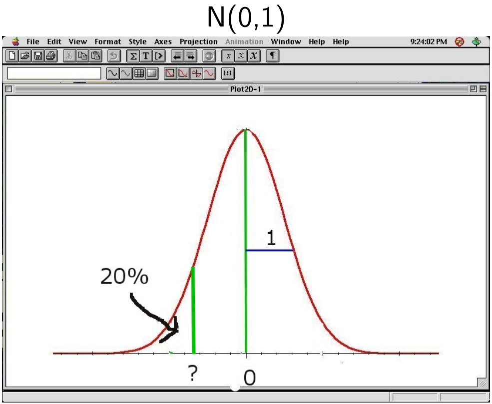
N(0,1)



Mileage Example

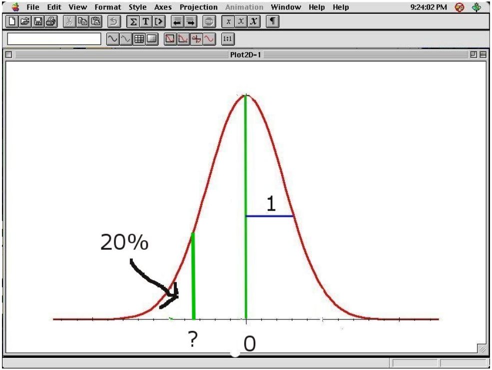
Worst 20% of cars?



Mileage Example

Worst 20% of cars?

N(0,1)



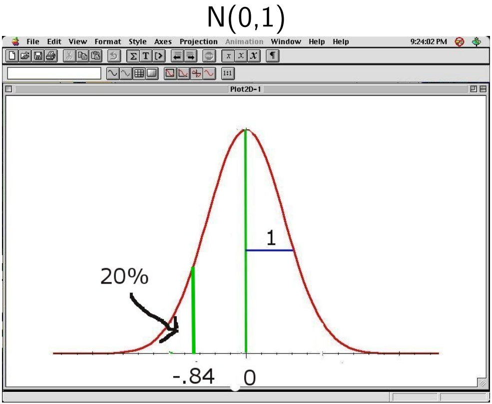
$P(Z < ?) = .20$

Table Z suggests $-.84$. to 2 decimal places.

$P(Z < -.84) = .2005$.

Mileage Example

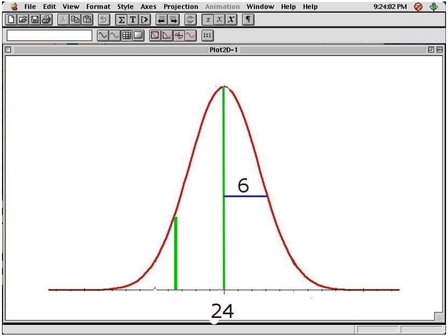
Worst 20% of cars?



Mileage Example

Worst 20% of cars?

Translating to $N(24, 6)$:

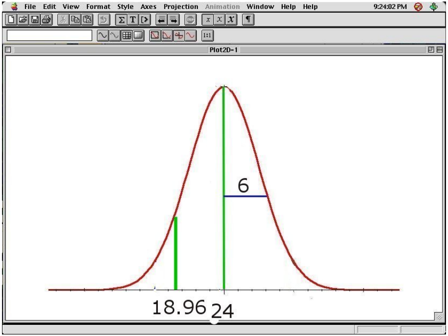


Having a Z score of $-.84$ means
being .84 std. dev. to the left of 24.
So the value is $24 + (-.84)(6) = 24 - 5.04 = 18.96$.

Mileage Example

Worst 20% of cars?

$N(24, 6)$:



The worst 20% cars are those with mileage below 18.96 mpg.

Mileage Example

Worst 20% of cars?

Instead of viewing

$$x = 24 + (-.84)(6)$$

as being clear from the picture, some people prefer to solve

$$\frac{x - 24}{6} = -.84.$$

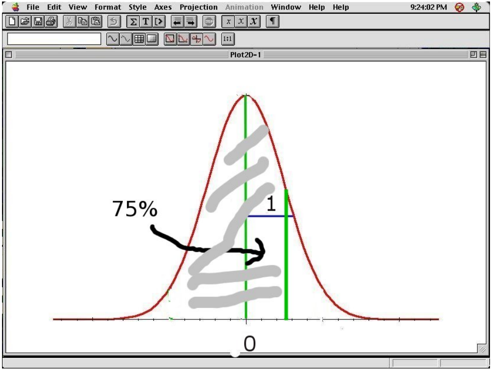
Mileage Example

Third Quartile?

Third Quartile?

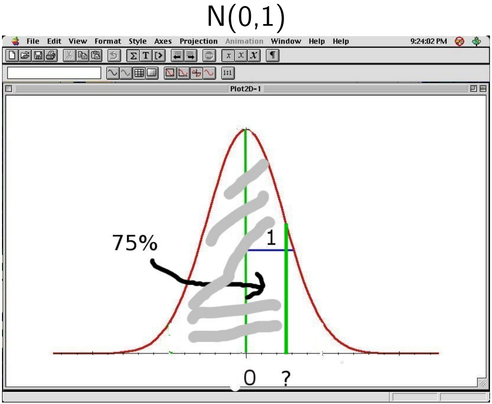
Again start with the $N(0,1)$ picture.

N(0,1)



Third Quartile?

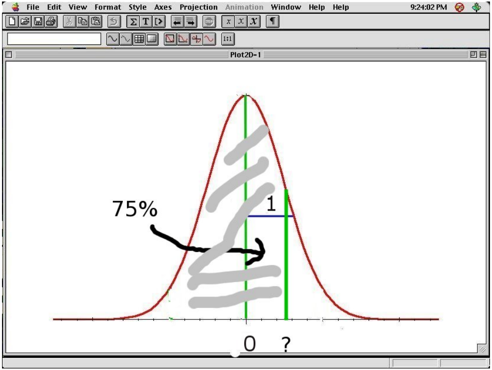
Again start with the $N(0, 1)$ picture.



Mileage Example

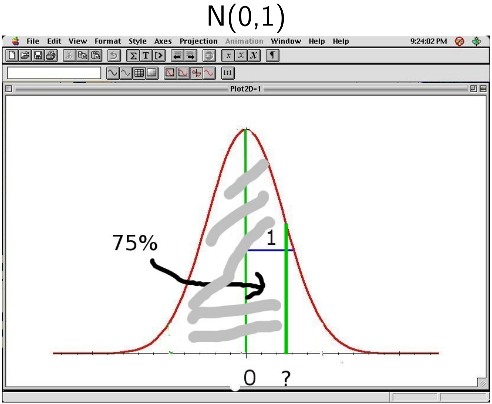
Third Quartile?

N(0,1)



Mileage Example

Third Quartile?



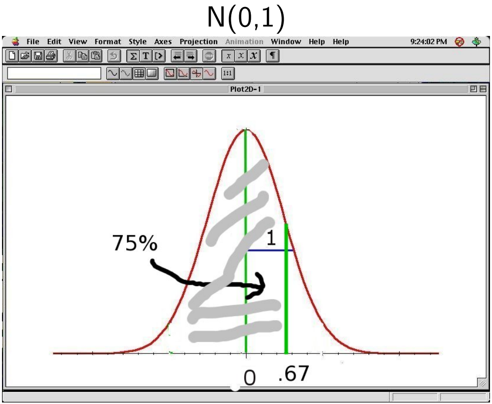
$P(Z < ?) = .75$

Table Z suggests .67. to 2 decimal places.

$P(Z < .67) = .7486.$

Mileage Example

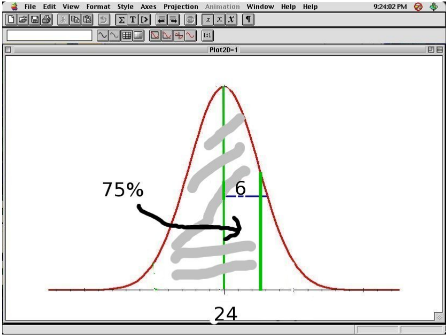
Third Quartile?



Mileage Example

Third Quartile?

Translating to $N(24, 6)$:

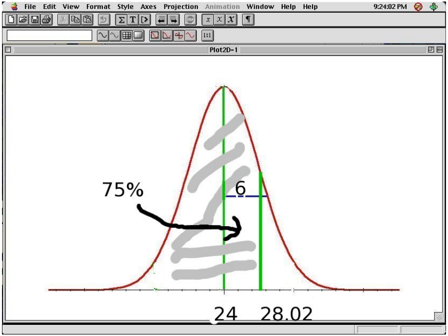


Having a Z score of .67 means being .67 std. dev. to the right of 24. So the value is $24 + (.67)(6) = 28.02$.

Mileage Example

Third Quartile?

$N(24, 6)$:



The 3rd quartile of cars are those with a mileage of 28.02 mpg.

Mileage Example

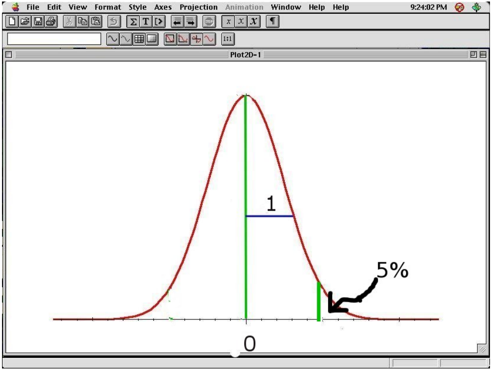
Gas mileage of the 5% most efficient?

Mileage Example

Gas mileage of the 5% most efficient?

Again start with the $N(0,1)$ picture.

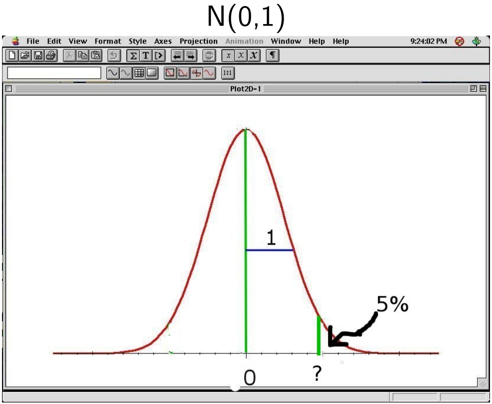
N(0,1)



Mileage Example

Gas mileage of the 5% most efficient?

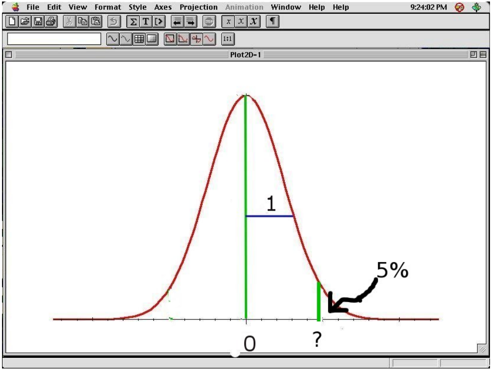
Again start with the $N(0,1)$ picture.



Mileage Example

Gas mileage of the 5% most efficient?

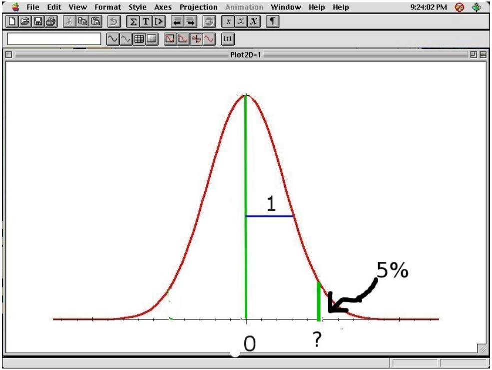
N(0,1)



Mileage Example

Gas mileage of the 5% most efficient?

N(0,1)



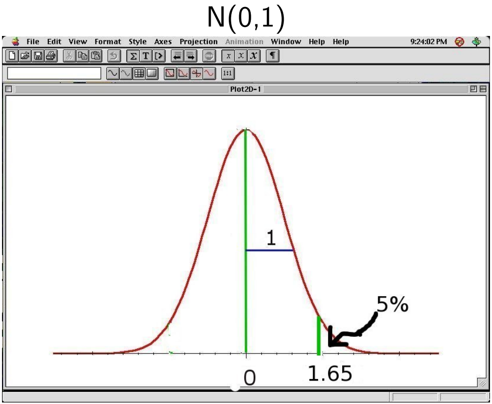
$P(Z > ?) = .05$. That means $P(Z < ?) = .95$.

Table Z suggests 1.65. to 2 decimal places.

$P(Z < 1.65) = .9505$. (1.64 is an equally good choice.)

Mileage Example

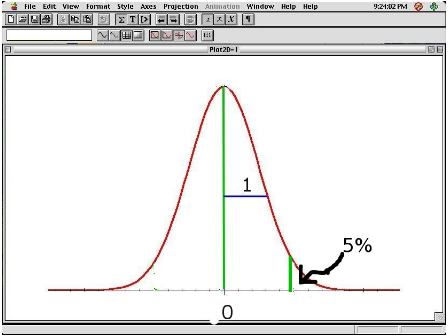
Gas mileage of the 5% most efficient?



Mileage Example

Gas mileage of the 5% most efficient?

Translating to $N(24, 6)$:

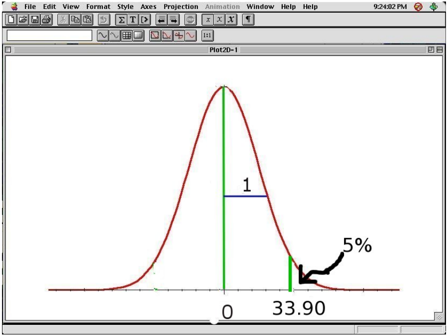


Having a Z score of 1.65 means being 1.65 std. dev. to the right of 24. So the value is $24 + (1.65)(6) = 33.90$.

Mileage Example

Gas mileage of the 5% most efficient?

$N(24, 6)$:



The top 5% of cars are those with a mileage of at least 22.90 mpg.