Math 1710 Class 10

Normal Distributions and Approximation

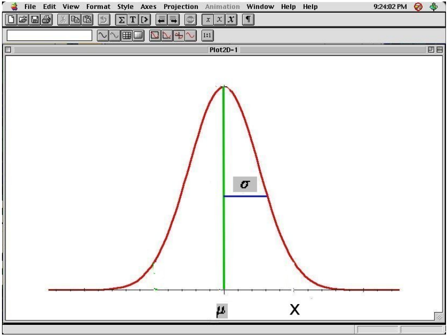
Dr. Back

Sep. 18, 2009

Z-scores

A general normal distribution:

$N(\mu, \sigma)$



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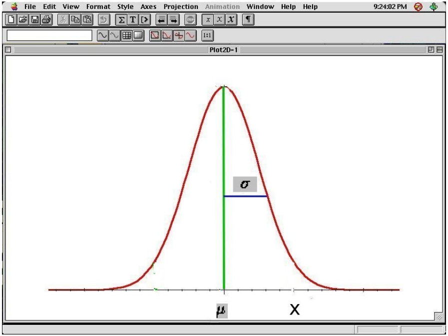

Can convert from a general $N(\mu, \sigma)$ to $N(0, 1)$ via the Z-score.

$$z = \frac{x - \mu}{\sigma}$$

UsingTable Z

For example $P(-.67 < Z < 1) = ?$

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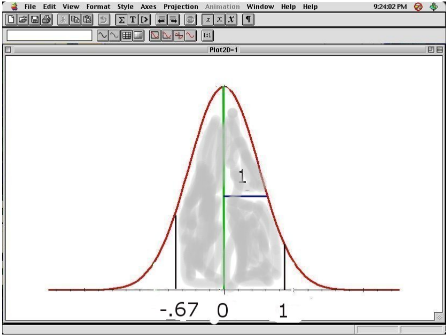
N(0,1)



Using Table Z

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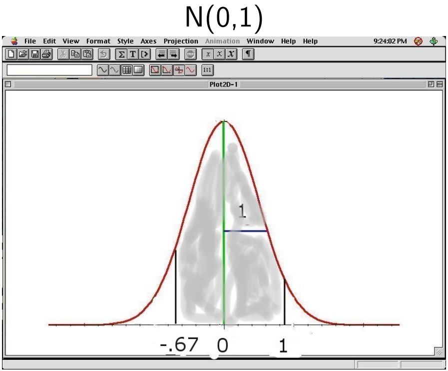

Table Z tells us $P(-.67 < Z < 1) = .8413 - .2514 = .5899.$

Success/Failure Condition

When n is large, binomial distributions can usually be approximated fairly well by normal ones.

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An outline of the case $p = .5$ is at the end of this lecture.

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You poll 100 people.

What is the probability that between 60 and 65 report approval?

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We can approximate $P(60 \leq X \leq 65)$ by $P(60 \leq Y \leq 65)$.
The Z score of 60 is $\frac{-10}{4.58} = -2.18$.
The Z score of 65 is $\frac{-5}{4.58} = -1.09$.
So $P(60 \leq Y \leq 65) = P(Z < -1.09) - P(Z < -2.18) = .1379 - .0146 = .1233$.

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So .1233 is not that good an approximation.

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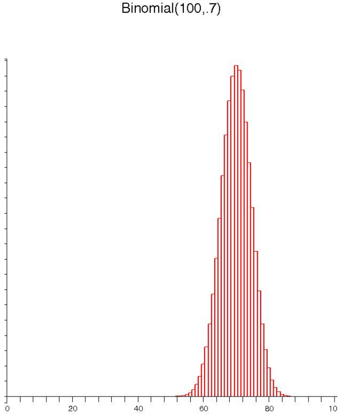
$P(60 < X < 65) = .09509$. would also be approximated by .1233!

Normal Approximation of X=Binomial(100,.7)

For each $x = 0, 1, 2, \ldots, 100$,
a rectangle with base 1 and height P(X=x) is drawn:



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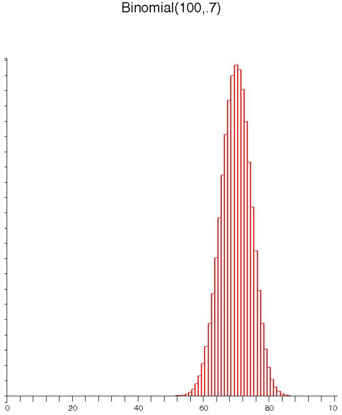
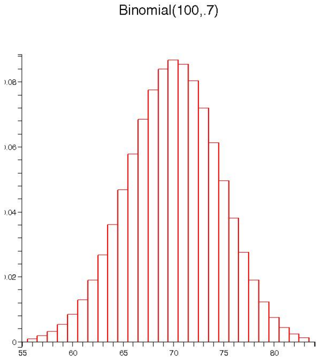


This makes the area of each bar a probability!

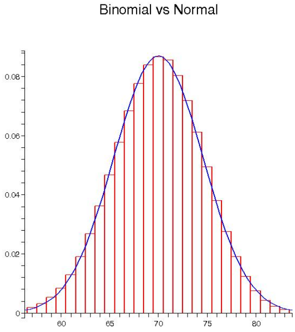
Normal Approximation of X=Binomial(100,.7)

Most of the action between 55 and 85.
Look at that part:



Normal Approximation of X=Binomial(100,.7)

With N(70,4.58) superimposed.



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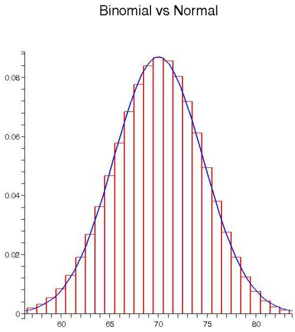
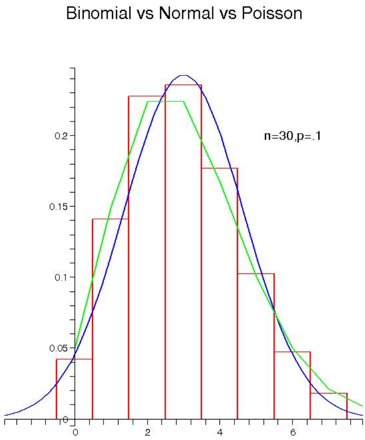


This is the good case when normal approximation works.

Normal Approximation of X=Binomial(30,.1)



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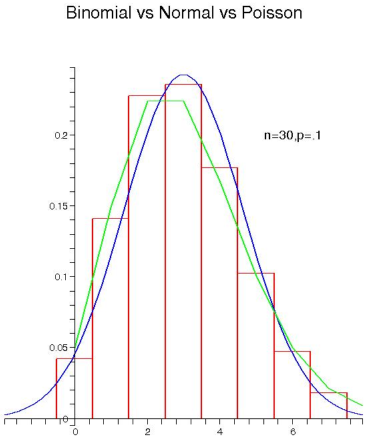

$\mu = 3$ and $\sigma = \sqrt{30(.1)(.9} = 1.64.$

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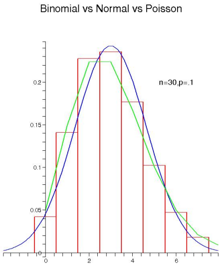

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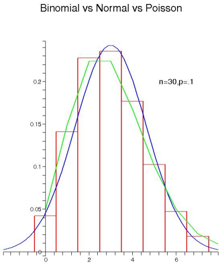
Two standard deviations to the left of the mean and our normal model is reporting negative values for the number of successes!

Normal Approximation of X=Binomial(30,.1)



$\mu = 3$ and $\sigma = \sqrt{30(.1)(.9} = 1.64$.

This is typical of what happens when success/failure is not satisfied.

$X = Binomial(100, .7)$ and $Y = N(70, 4.58)$.

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The problem is that for a continuous model like Y,
$P(Y = 65) = 0$.
But $P(X = 65) = .04678$.

$X = Binomial(100, .7)$ and $Y = N(70, 4.58)$.

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So Y doesn't care about $<$ vs. $\leq$. But X does!

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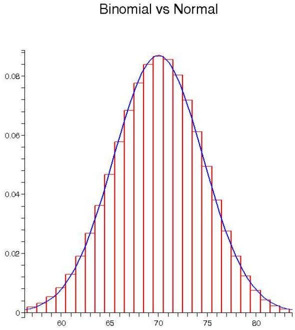
Looking closely at the picture provides the key:



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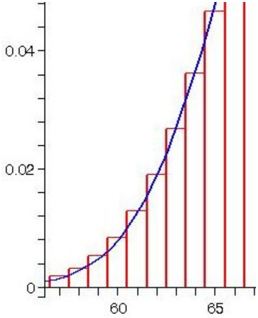

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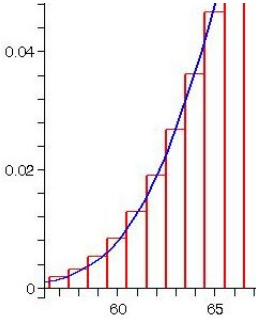

It is really the area under the normal curve between 64.5 and 65.5 which approximates P(X=65).

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The z-scores of 59.5 and 65.5 are
$\frac{-10.5}{4.58} = -2.29$ and $\frac{-4.5}{4.58} = -.98$ resp.

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So $P(59.5 \leq Y \leq 65.5) = P(-2.29 \leq Z \leq -.98) = .1635 - .0110 = .1525.$
Much closer to $P(60 \leq X \leq 65) = .15036$!

$X = Binomial(100, .7)$ and $Y = N(70, 4.58)$.

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So $P(60 \leq X \leq 65) = .15036$ should be approximated by $P(59.5 \leq Y \leq 65.5)$.
Similarly $P(60.5 \leq Y \leq 64.5)$
$= P(Z < -1.20) - P(Z < -2.07) = .1151 - .0192 = .0959$ is a great approximation to $P(60 < X < 65) = .09509$.

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So $P(60 \leq X \leq 65) = .15036$ should be approximated by $P(59.5 \leq Y \leq 65.5)$.
This is called the continuity approximation.
You needn't use it on exams or homework!
(We suggest you not.)

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unimodal and symmetric distribution.

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Here's a symmetric and unimodal example to illustrate this:

Nearly Normal Condition

n=318: -10,-9,-8, . . .-2, 100 -1's, 100 0's, . . .

tl+mid

Frequency breakdown of tl+mid

No Selector

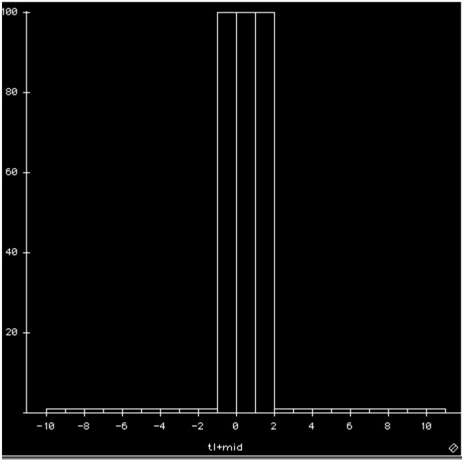
Total Cases 318

Number of Categories 21

Group	Count
-10	1
-9	1
-8	1
-7	1
-6	1
-5	1
-4	1
-3	1
-2	1
-1	100
0	100
1	100
2	1
3	1
4	1
5	1
6	1
7	1
8	1
9	1
10	1

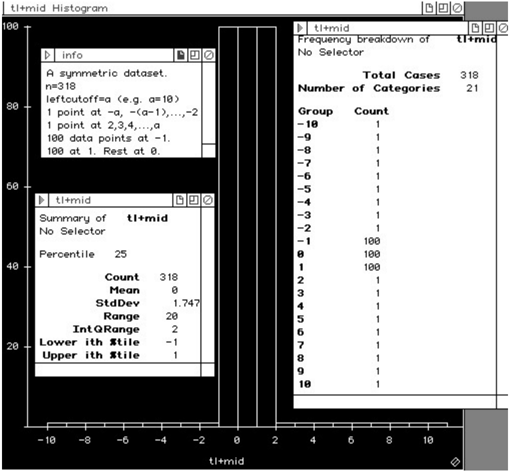
Nearly Normal Condition

A Symmetric and Unimodal Histogram



Nearly Normal Condition

Summaries



Nearly Normal Condition

Summary Statistics: $n = 318$, $\bar{x} = 0$, $s = 1.747$

-10,-9,-8, . . . -2, 100 -1's, 100 0's, . . .

tl+mid

Summary of **tl+mid**
No Selector

Percentile 25

Count	318
Mean	0
StdDev	1.747
Range	20
IntQRange	2
Lower ith %tile	-1
Upper ith %tile	1

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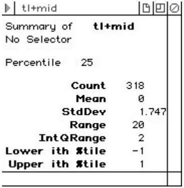

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Only $18 = 2 \cdot 9$ of 318 values
are more than 1 std. dev. (1.747) from $\bar{x} = 0$.

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94% within 1 std dev vs. 68% for normal!

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94% within 1 std dev vs. 68% for normal!
4^{+}% more than 3 std dev vs. 0.3% for normal!

Nearly Normal Condition

Your textbook uses this as a synonym for a

unimodal and symmetric distribution.

This doesn't mean unimodal and symmetric distributions are necessarily well approximated by normal ones.

It is true that unimodal and symmetric does mean normality might be at least roughly indicative.

Some statistical methods derived under assumptions of normality can be shown to usually work more generally. I think this is what the authors really have in mind.

Chapter 17 #31 (Lefties)

- 200 seats
- 30 for left-handers
- class size 188
- on avg., 13% left handers

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Model num *left* hnd students by X=Binomial(188,.13).

Chapter 17 #31 (Lefties)

- 200 seats
- 30 for left-handers
- class size 188
- on avg., 13% left handers

Prob. run out of right hand seats?

Model num *left* hnd students by X=Binomial(188,.13).

$\mu = 188 \cdot .13 = 24.44$, $\sigma = \sqrt{188 \cdot .13 \cdot .87} = 4.61$.

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- class size 188
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Model num *left* hnd students by X=Binomial(188,.13).

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Approximate by $Y = N(24.44, 4.61)$.

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Model num *left* hnd students by X=Binomial(188,.13).

$\mu = 188 \cdot .13 = 24.44$, $\sigma = \sqrt{188 \cdot .13 \cdot .87} = 4.61$.

Approximate by $Y = N(24.44, 4.61)$.

Calculate $P(Y < 18)$.

Chapter 17 #33 (Telemarketers)

- Story: 12% success rate on calls.
- Experience: 10 successes out of 200 calls.
- (first week)

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Likely Misled?

Model by X=Binomial(200,.12)

Reasonable?

Chapter 17 #33 (Telemarketers)

- Story: 12% success rate on calls.
- Experience: 10 successes out of 200 calls.
- (first week)

Likely Misled?

Model by X=Binomial(200,.12)

$\mu = 24$, $\sigma = \sqrt{200 \cdot .12 \cdot .88} = 4.60$.

Chapter 17 #33 (Telemarketers)

- Story: 12% success rate on calls.
- Experience: 10 successes out of 200 calls.
- (first week)

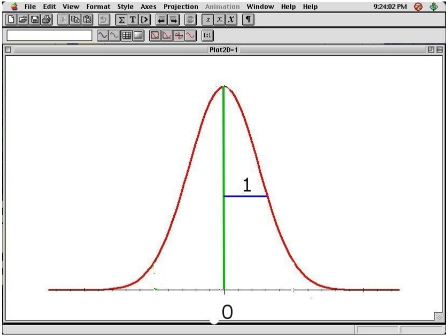
Likely Misled?
Model by X=Binomial(200,.12)
$\mu = 24$, $\sigma = \sqrt{200 \cdot .12 \cdot .88} = 4.60$.
10 is more than 3 standard deviations below the mean, so not very likely.
So model (story) likely inappropriate.

Chapter 17 #33 (Telemarketers)

- Story: 12% success rate on calls.
- Experience: 10 successes out of 200 calls.
- (first week)

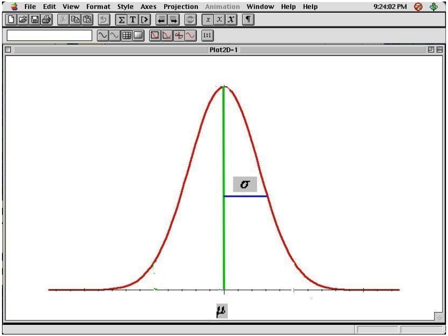
Likely Misled?
Model by X=Binomial(200,.12)
$\mu = 24$, $\sigma = \sqrt{200 \cdot .12 \cdot .88} = 4.60$.
So model (story) likely inappropriate.
But for 1000 salesmen starting the same week, it wouldn't be surprising for one to have this experience even if the model is right.

Normal Distribution Formula

N(0,1) 

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

Normal Distribution Formula

$N(\mu, \sigma)$ 

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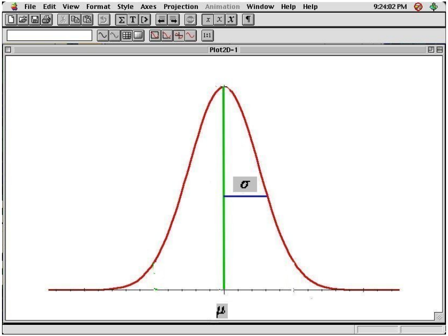
$N(\mu, \sigma)$ 

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These formulas and the following argument are far above the basic level of our course.

Why Normal Out of Binomial?

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$\mu = m$ and $\sigma = \sqrt{.5m}$.

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Strategy: Compare a_k to a_0 using the approximation $\ln(1 + x) \sim x$ for x small.

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So using $\ln(1+x) \sim x$,

$$\ln a_k \sim \ln a_0 - 2\left(\frac{1}{m} + \frac{2}{m} + \ldots + \frac{k-1}{m}\right) - \frac{k}{m}$$

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as desired.

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