Math 1710 Class 3

Conditional Probability, Tree Diagrams

Dr. Back

Sep. 2, 2009

Definition

"Probability of A given B."

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

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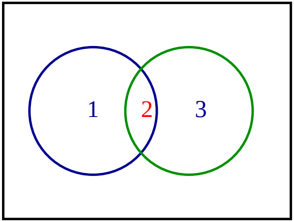

$$A = \{1, 2\} \quad B = \{2, 3\}$$

$$A \cap B = \{2\}$$

Independence

A and B are independent events if

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10% have oil leaks
1% both.
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$$P(sticky|leaky) = \frac{P(both)}{P(leaky)} = \frac{.01}{.10} = .10 = P(sticky).$$

So they are independent.

Independence

A and B are independent events if

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Box has 2 white tickets labeled 1
2 black labeled 1
1 white labeled 8
1 black one labeled 6.
Are drawing a 1 and color black independent?

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Are drawing a 1 and color black independent?

$$P(1|black) = \frac{2}{3} \qquad P(1) = \frac{4}{6}$$

Equal, so they are independent.

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2 black labeled 1
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Are drawing an 8 and color black independent?

$$P(8|black) = \frac{0}{3} \qquad P(8) = \frac{1}{6}$$

Not equal, so not independent.

Collins Case

Purse Snatching Suspects:

Yellow Auto:	$1/10$
Man with Mustache:	$1/4$
Woman with Ponytail:	$1/10$
Woman with Blond Hair:	$1/3$
Black Man with Beard:	$1/10$
Interracial Couple in Car:	$1/1000$

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1/12,000,000 chance any couple matches these.

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Defendants convicted.

Reversed on appeal.

Prosecutor's Fallacy

Just because

$$P(\text{DNA evidence} \mid \text{innocent})$$

is small, it doesn't follow that

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"The chance that anyone else but the appellant left the hairs at the scene of the crime is 6 billion to 1." (1991)

Defendant's Fallacy

1 in a million match this DNA

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1 in a million match this DNA

So there are hundreds of matches
and only a small chance the defendant is guilty.

From a 2 way table

	Jeans	**Other**
Male	12	5
Female	8	11

From a 2 way table

With Marginal Distributions:

	Jeans	Other	
Male	12	5	17
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	20	16	36

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P(M) ?

$$P(M) = \frac{17}{36}$$

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P(Male wears jeans)?

i.e. $P(J|M)$

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$$P(J|M) = \frac{P(J \cap M)}{P(M)}$$

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P(Male wears jeans)?
i.e. $P(J|M)$

$$P(J|M) = \frac{P(J \cap M)}{P(M)} = \frac{12}{17}$$

More precisely:

$$\frac{12}{36} \Big/ \frac{17}{36}$$

From a 2 way table

With Marginal Distributions:

	Jeans	**Other**	
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P(Someone wearing jeans is male)?
i.e. $P(M|J)$

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Are gender and wearing jeans disjoint? independent?

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Are gender and wearing jeans disjoint? independent?

$$P(M \cap J) > 0$$

So they sometimes both happen.
Not disjoint.

From a 2 way table

With Marginal Distributions:

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Are gender and wearing jeans disjoint? independent?

$$P(M|J) \neq P(M)$$

So not independent.

Most Pairs of Events are Neither!

Checking disjointness doesn't even require probabilities.

Checking independence does.

But when events are related in one of these ways, it is worth noting.

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Sps:
A=getting swine flu
B=getting a shot.

What would disjoint/independent mean here?

Disjointness and Independence Never Happen Together (almost)

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This holds as long as none of the events have probability 0.

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For if A and B are disjoint,

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = 0 \neq P(A).$$

So A and B are not independent.

Disjointness and Independence Never Happen Together (almost)

This holds as long as none of the events have probability 0.

For if A and B are disjoint,

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = 0 \neq P(A).$$

So A and B are not independent.

(Events of probability 0 do sometimes happen; e.g. a person exactly 173 cm. tall.)

Ch. 16 #21 Variant

Two games against same opponent.
P(win 1st)=.4
If you win 1st, P(win 2nd)=.2
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Sequences of events

Tree diagrams are good for keeping track of all the combinations in multi-stage events.

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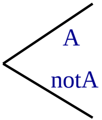
First Step:

A



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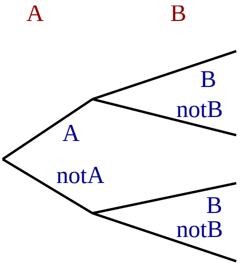
Second Step:



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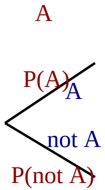

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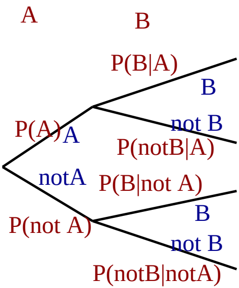

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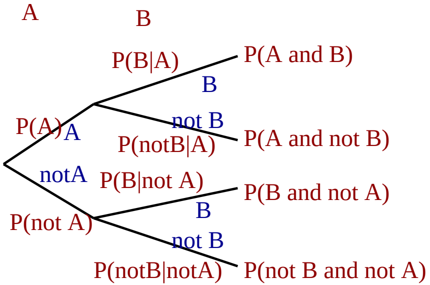
Graphical version of the mult rule $P(A \cap B) = P(B|A)P(A)$.
Final After Multiplying:



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Solution: A=win 1st, B=win 2nd

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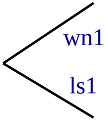

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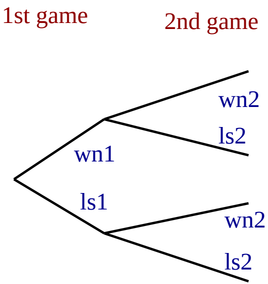

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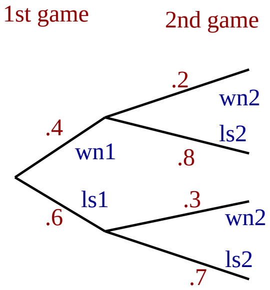

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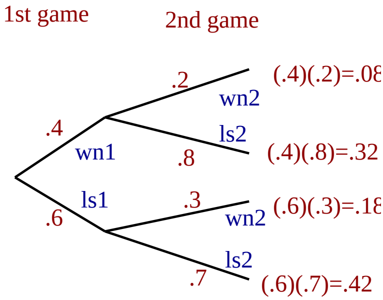

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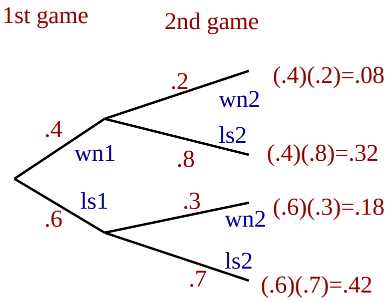

Note:
.08+.32+.18+.42=1
since they count all
the disjoint possibilitie
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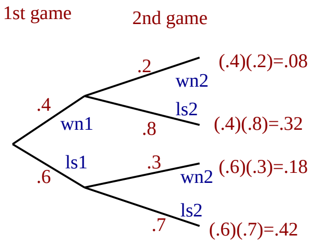

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P(wn 1 of 2)=.32+.18=.5

P(wn 2nd) = .08+.18=.26

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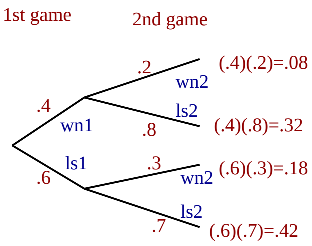

Note:
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P(wn 1 of 2)=.32+.18=.50

P(wn 2nd) = .08+.18=.26

P(wn 1st|wn 2nd)=
P(wn1st&wn2nd)/P(wn2nd)
=.08/.26=.308