

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

Math 1710 Class 11

Normal Distributions, Approximation
Dr. Back

Sep. 21, 2009

Wed Mileage Example

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

Suppose a normal model $N(24 \text{ mpg}, 6 \text{ mpg})$ describes fuel efficiency of cars in a region:

Wed Mileage Example

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

Gas mileage of the middle 30%?

Wed Mileage Example

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

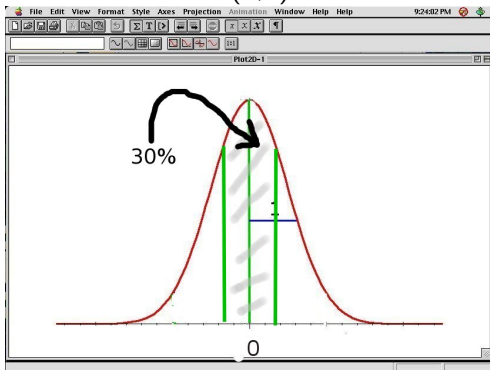
A Random
Variable
Problem

Proof of
Normal
Approximation

Gas mileage of the middle 30%?

Again start with the $N(0, 1)$ picture.

$N(0, 1)$

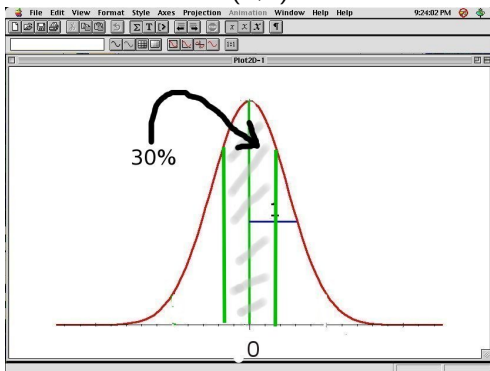


Wed Mileage Example

Gas mileage of the middle 30%?

Again start with the $N(0, 1)$ picture.

$N(0, 1)$



Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

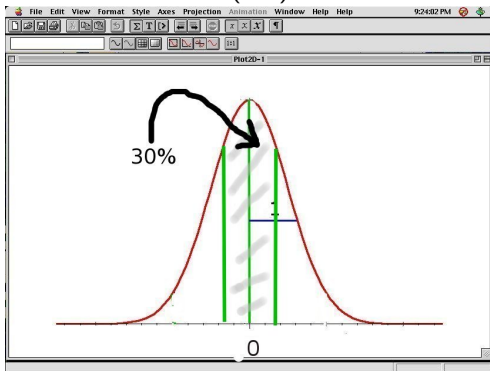
A Random
Variable
Problem

Proof of
Normal
Approximation

Wed Mileage Example

Gas mileage of the middle 30%?

$N(0,1)$



The only trick here is what $P(Z < ?)$ should be.

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

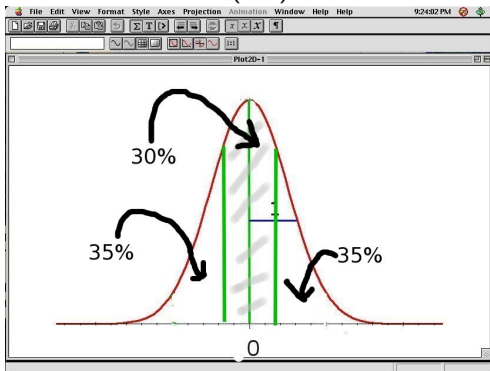
A Random
Variable
Problem

Proof of
Normal
Approximation

Wed Mileage Example

Gas mileage of the middle 30%?

$N(0,1)$



$P(Z < ?) = .35 + .30 = .65$. NOT $P(Z < ?) = .30$

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

Wed Mileage Example

Gas mileage of the middle 30%?

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

$N(0,1)$

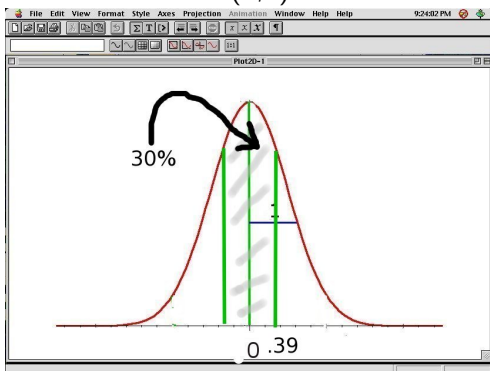


Table Z suggests .39. to 2 decimal places.

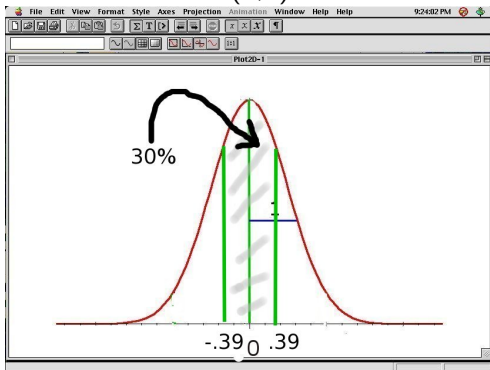
$$P(Z < .39) = .6517.$$

Wed Mileage Example

Gas mileage of the middle 30%?

By symmetry:

$N(0,1)$



Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

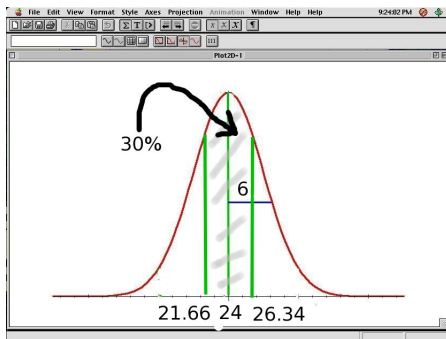
A Random
Variable
Problem

Proof of
Normal
Approximation

Wed Mileage Example

Gas mileage of the middle 30%?

Translating to $N(24, 6)$:



Having a Z score of .39 means
being .39 std. dev. to the right of 24.
So the uppervalue is $24 + (.39)(6) = 26.34$.

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

Wed Mileage Example

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

Gas mileage of the middle 30%?

Having a Z score of $-.39$ means
being $.39$ std. dev. to the left of 24.

So the lower value is $24 - (.39)(6) = 21.66$.

Wed Mileage Example

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

Gas mileage of the middle 30%?

The middle 30% of cars are those with a mileage between 21.66 and 26.34 mpg.

Chapter 17 #31 (Lefties)

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

- 200 seats
- 30 for left-handers
- class size 188
- on avg., 13% left handers

Chapter 17 #31 (Lefties)

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

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Normal
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Prob. run out of right hand seats?

Chapter 17 #31 (Lefties)

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

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Normal
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- 200 seats
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- class size 188
- on avg., 13% left handers

Prob. run out of right hand seats?

Model num *left* hnd students by $X = \text{Binomial}(188, .13)$.

Chapter 17 #31 (Lefties)

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

- 200 seats
- 30 for left-handers
- class size 188
- on avg., 13% left handers

Prob. run out of right hand seats?

Model num *left* hnd students by $X = \text{Binomial}(188, .13)$.

$$\mu = 188 \cdot .13 = 24.44, \sigma = \sqrt{188 \cdot .13 \cdot .87} = 4.61.$$

Chapter 17 #31 (Lefties)

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

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Normal
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- 200 seats
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- class size 188
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Prob. run out of right hand seats?

Model num *left* hnd students by $X = \text{Binomial}(188, .13)$.

$$\mu = 188 \cdot .13 = 24.44, \sigma = \sqrt{188 \cdot .13 \cdot .87} = 4.61.$$

Approximate by $Y = N(24.44, 4.61)$.

Chapter 17 #31 (Lefties)

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

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Normal
Approximation

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$$\mu = 188 \cdot .13 = 24.44, \sigma = \sqrt{188 \cdot .13 \cdot .87} = 4.61.$$

Approximate by $Y = N(24.44, 4.61)$.

Calculate $P(Y < 18)$.

Chapter 17 #31 (Lefties)

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

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Normal
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Prob. run out of right hand seats?

Model num *left* hnd students by $X = \text{Binomial}(188, .13)$.

Approximate by $Y = N(24.44, 4.61)$.

Calculate $P(Y < 18)$.

The z-score of 18 is

$$\frac{18 - 24.44}{4.61} = -1.40.$$

$$P(Z < -1.40) = .0808.$$

Chapter 17 #31 (Lefties)

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
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Prob. run out of right hand seats?

Model num *left* hnd students by $X = \text{Binomial}(188, .13)$.

Approximate by $Y = N(24.44, 4.61)$.

Calculate $P(Y < 18)$.

The z-score of 18 is

$$\frac{18 - 24.44}{4.61} = -1.40.$$

$$P(Z < -1.40) = .0808.$$

You could instead calculate $P(Y \leq 17)$ leading to

$$P(Z < -1.61) = .0537.$$

Chapter 17 #31 (Lefties)

- 200 seats
- 30 for left-handers
- class size 188
- on avg., 13% left handers

Prob. run out of right hand seats?

Model num *left* hnd students by $X = \text{Binomial}(188, .13)$.

Approximate by $Y = N(24.44, 4.61)$.

Calculate $P(Y < 18)$.

The z-score of 18 is

$$\frac{18 - 24.44}{4.61} = -1.40.$$

$$P(Z < -1.40) = .0808.$$

You could instead calculate $P(Y \leq 17)$ leading to

$$P(Z < -1.61) = .0537.$$

With the continuity correction, you'd split the difference.

Either of these would be $P(Z < -1.51) = .0655$.

Chapter 17 #33 (Telemarketers)

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

- Story: 12% success rate on calls.
- Experience: 10 successes out of 200 calls.
- (first week)

Chapter 17 #33 (Telemarketers)

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

- Story: 12% success rate on calls.
- Experience: 10 successes out of 200 calls.
- (first week)

Likely Misled?

Chapter 17 #33 (Telemarketers)

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

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Normal
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Chapter 17 #33 (Telemarketers)

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

- Story: 12% success rate on calls.
- Experience: 10 successes out of 200 calls.
- (first week)

Likely Misled?

Model by $X = \text{Binomial}(200, .12)$

Reasonable?

Chapter 17 #33 (Telemarketers)

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

- Story: 12% success rate on calls.
- Experience: 10 successes out of 200 calls.
- (first week)

Likely Misled?

Model by $X = \text{Binomial}(200, .12)$

$$\mu = 24, \sigma = \sqrt{200 \cdot .12 \cdot .88} = 4.60.$$

Chapter 17 #33 (Telemarketers)

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

- Story: 12% success rate on calls.
- Experience: 10 successes out of 200 calls.
- (first week)

Likely Misled?

Model by $X = \text{Binomial}(200, .12)$

$$\mu = 24, \sigma = \sqrt{200 \cdot .12 \cdot .88} = 4.60.$$

10 is more than 3 standard deviations below the mean, so not very likely.

So model (story) likely inappropriate.

Chapter 17 #33 (Telemarketers)

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

- Story: 12% success rate on calls.
- Experience: 10 successes out of 200 calls.
- (first week)

Likely Misled?

Model by $X = \text{Binomial}(200, .12)$

$$\mu = 24, \sigma = \sqrt{200 \cdot .12 \cdot .88} = 4.60.$$

So model (story) likely inappropriate.

But for 1000 salesmen starting the same week, it wouldn't be surprising for one to have this experience even if the model is right.

Chapter 6 #43 (Egg Weights)

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

- Normal Model
- Desired min 54 gm.

Chapter 6 #43 (Egg Weights)

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

(a) Earlier $\mu = 50.9$, 28% above min. $\sigma = ?$

Chapter 6 #43 (Egg Weights)

Math 1710
Class 11

V2c

Last Time

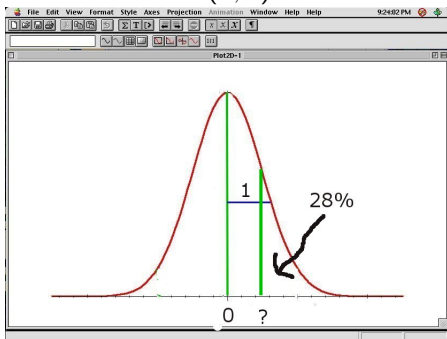
More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

(a) Earlier $\mu = 50.9$, 28% above min. $\sigma = ?$

$N(0, 1)$



Chapter 6 #43 (Egg Weights)

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

$$P(Z < .58) = .7190$$

Chapter 6 #43 (Egg Weights)

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

$$\frac{54 - 50.9}{\sigma} = .58.$$

$$\text{So } \sigma = \frac{3.1}{.058} = 5.34$$

Chapter 6 #43 (Egg Weights)

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

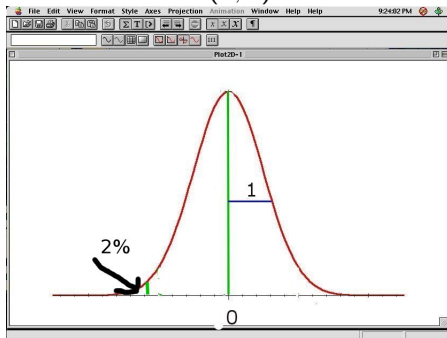
Proof of
Normal
Approximation

(b) Later $\mu = 67.1$, 98% above min. $\sigma = ?$

Chapter 6 #43 (Egg Weights)

(b) Later $\mu = 67.1$, 98% above min. $\sigma = ?$

$N(0, 1)$



Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

Chapter 6 #43 (Egg Weights)

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

(c) Which are more consistent?

Chapter 6 #43 (Egg Weights)

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

(c) Which are more consistent?

Just compare values of σ in (a) and (b)

Chapter 6 #43 (Egg Weights)

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

(d) (in earlier edition) 8% underweight and 12% over 70 gm.

$\mu, \sigma = ?$

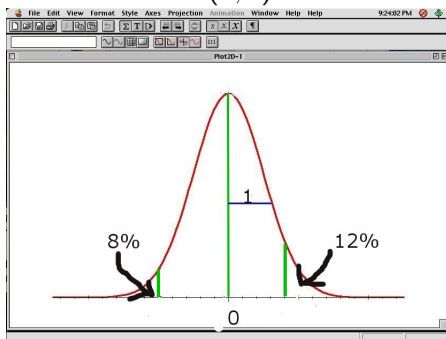
Chapter 6 #43 (Egg Weights)

(d) (in earlier edition) 8% underweight and 12% over 70 gm.

$\mu, \sigma = ?$

Find z-scores with $P(Z < z_1) = .08$ and $P(Z > z_2) = .12$

$N(0, 1)$



Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

Chapter 6 #43 (Egg Weights)

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

(d) (in earlier edition) 8% underweight and 12% over 70 gm.

$\mu, \sigma = ?$

Find z-scores with $P(Z < z_1) = .08$ and $P(Z > z_2) = .12$

Then $70 - 54 = (z_2 - z_1)\sigma$.

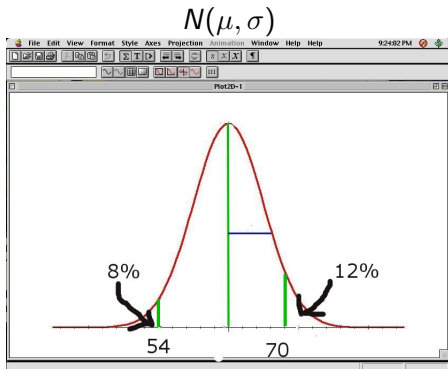
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Find z-scores with $P(Z < z_1) = .08$ and $P(Z > z_2) = .12$

Then $70 - 54 = (z_2 - z_1)\sigma$.



Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

Chapter 6 #43 (Egg Weights)

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

(d) (in earlier edition) 8% underweight and 12% over 70 gm.

$\mu, \sigma = ?$

Find z-scores with $P(Z < z_1) = .08$ and $P(Z > z_2) = .12$

Then $70 - 54 = (z_2 - z_1)\sigma$.

Knowing σ , find μ from e.g. z-score of 54 is z_1 .

RV Review Problem #5

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

In problem #3, suppose the amounts of cereal in both size bowls are normally distributed. ($N(1.5, .3)$ and $N(2.5, .4)$.)

You pour a small and a large bowl.

- (a) What is the probability you poured out more than 4.5 ounces of cereal in the two bowls together.
- (b) What is the probability the small bowl contains more cereal than the large one?

RV Review Problem #5

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

- small bowls $N(1.5, .3)$
- large bowls $N(2.5, .4)$
- one small, one large bowl

$P(\text{total} > 4.5)?$

RV Review Problem #5

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

- small bowls $N(1.5, .3)$
- large bowls $N(2.5, .4)$
- one small, one large bowl

$P(\text{total} > 4.5)?$

Solution: First define some RV's:

RV Review Problem #5

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

- small bowls $N(1.5, .3)$
- large bowls $N(2.5, .4)$
- one small, one large bowl

$P(\text{total} > 4.5)?$

Solution: First define some RV's:

Let S be an RV describing amt. of cereal in a small bowl.

RV Review Problem #5

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

- small bowls $N(1.5, .3)$
- large bowls $N(2.5, .4)$
- one small, one large bowl

$P(\text{total} > 4.5)?$

Solution: First define some RV's:

Let S be an RV describing amt. of cereal in a small bowl.

Let L be an RV describing amt. of cereal in a lg. bowl.

RV Review Problem #5

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

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- large bowls $N(2.5, .4)$
- one small, one large bowl

$P(\text{total} > 4.5)?$

Solution: First define some RV's:

Let S be an RV describing amt. of cereal in a small bowl.

Let L be an RV describing amt. of cereal in a lg. bowl.

Let T be an RV describing total amt. of cereal.

RV Review Problem #5

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

- small bowls $N(1.5, .3)$
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- one small, one large bowl

$P(\text{total} > 4.5)?$

Let S be an RV describing amt. of cereal in a small bowl.

Let L be an RV describing amt. of cereal in a lg. bowl.

Let T be an RV describing total amt. of cereal.

$$T = L + S$$

RV Review Problem #5

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

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- one small, one large bowl

$P(\text{total} > 4.5)?$

$$T = L + S$$

The sum of (indep.) normal RV's is again normal.

RV Review Problem #5

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

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$P(\text{total} > 4.5)?$

$$T = L + S$$

The sum of (indep.) normal RV's is again normal.

So what we need are: $E(T)$? $\text{Var}(T)$? σ_T ?

RV Review Problem #5

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

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$P(\text{total} > 4.5)?$

$$T = L + S$$

The sum of (indep.) normal RV's is again normal.

So what we need are: $E(T)$? $\text{Var}(T)$? σ_T ?

$$E(T) = E(L) + E(S) = 1.5 + 2.5 = 4.$$

RV Review Problem #5

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

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$P(\text{total} > 4.5)?$

$$T = L + S$$

The sum of (indep.) normal RV's is again normal.

So what we need are: $E(T)$? $\text{Var}(T)$? σ_T ?

$$E(T) = E(L) + E(S) = 1.5 + 2.5 = 4.$$

Assuming independence

$$\text{Var}(T) = \text{Var}(L) + \text{Var}(S) = .4^2 + .3^2 = .25.$$

RV Review Problem #5

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

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- one small, one large bowl

$P(\text{total} > 4.5)?$

$$T = L + S$$

The sum of (indep.) normal RV's is again normal.

So what we need are: $E(T)$? $\text{Var}(T)$? σ_T ?

$$E(T) = E(L) + E(S) = 1.5 + 2.5 = 4.$$

Assuming independence

$$\text{Var}(T) = \text{Var}(L) + \text{Var}(S) = .4^2 + .3^2 = .25.$$

$$\sigma_T = \sqrt{.25} = .5$$

RV Review Problem #5

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

- small bowls $N(1.5, .3)$
- large bowls $N(2.5, .4)$
- one small, one large bowl

$P(\text{total} > 4.5)?$

$$T = L + S$$

The sum of (indep.) normal RV's is again normal.

So what we need are: $E(T)$? $\text{Var}(T)$? σ_T ?

$$E(T) = E(L) + E(S) = 1.5 + 2.5 = 4.$$

Assuming independence

$$\text{Var}(T) = \text{Var}(L) + \text{Var}(S) = .4^2 + .3^2 = .25.$$

$$\sigma_T = \sqrt{.25} = .5$$

Thus $T \sim N(4, .5)$.

RV Review Problem #5

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

- small bowls $N(1.5, .3)$
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$P(\text{total} > 4.5)?$

Thus $T \sim N(4, .5)$.

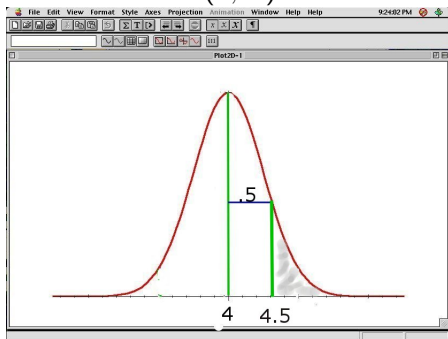
We need $P(T > 4.5)$:

RV Review Problem #5

Thus $T \sim N(4, .5)$.

We need $P(T > 4.5)$:

$N(4, .5)$



Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

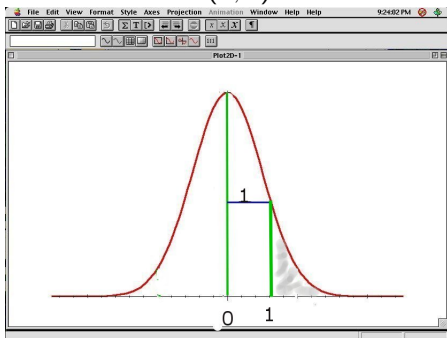
RV Review Problem #5

Thus $T \sim N(4, .5)$.

We need $P(T > 4.5)$:

The z score is 1 so

$N(0, 1)$



Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

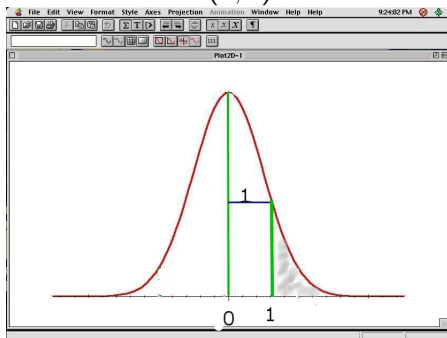
A Random
Variable
Problem

Proof of
Normal
Approximation

RV Review Problem #5

The z score is 1 so

$N(0, 1)$



And $P(Z > 1) = P(Z < -1) = .1587$.

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

RV Review Problem #5

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

In problem #3, suppose the amounts of cereal in both size bowls are normally distributed. ($N(1.5, .3)$ and $N(2.5, .4)$.) You pour a small and a large bowl.

- (b) What is the probability the small bowl contains more cereal than the large one?

RV Review Problem #5

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

In problem #3, suppose the amounts of cereal in both size bowls are normally distributed. ($N(1.5, .3)$ and $N(2.5, .4)$.) You pour a small and a large bowl.

(b) What is the probability the small bowl contains more cereal than the large one?

- small bowls $N(1.5, .3)$
- large bowls $N(2.5, .4)$
- one small, one large bowl

$P(S > L)$?

RV Review Problem #5

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

- small bowls $N(1.5, .3)$
- large bowls $N(2.5, .4)$
- one small, one large bowl

$P(S > L)?$

At first glance this is *hard!*

Two curves, how do you relate them?

RV Review Problem #5

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

- small bowls $N(1.5, .3)$
- large bowls $N(2.5, .4)$
- one small, one large bowl

$P(S > L)$?

Trick: Translate to $P(L - S < 0)$.

RV Review Problem #5

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

- small bowls $N(1.5, .3)$
- large bowls $N(2.5, .4)$
- one small, one large bowl

$P(S > L)?$

Trick: Translate to $P(L - S < 0)$.

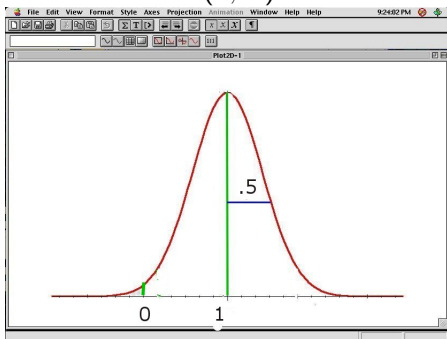
As before $L - S$ has mean 1 and std. dev. .5.

RV Review Problem #5

$$P(S > L)?$$

Trick: Translate to $P(L - S < 0)$.

$$N(1, .5)$$



Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

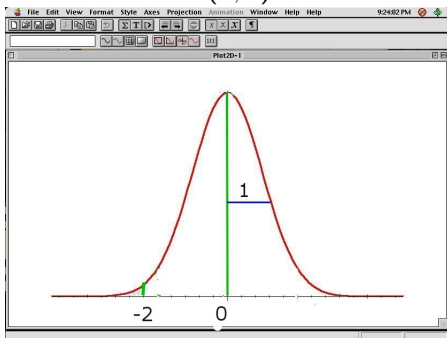
RV Review Problem #5

$P(S > L)$?

Trick: Translate to $P(L - S < 0)$.

The z score is -2 so

$N(0, 1)$



Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

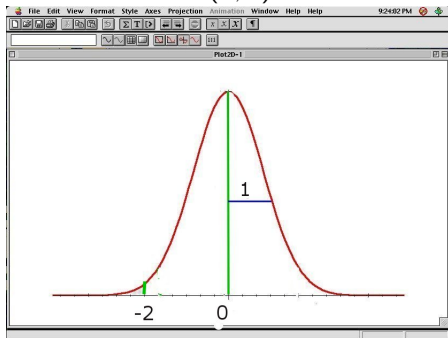
RV Review Problem #5

$P(S > L)$?

Trick: Translate to $P(L - S < 0)$.

The z score is -2 so

$N(0, 1)$



$P(Z < -2) = .0228.$

Normal Distribution Formula

Math 1710
Class 11

V2c

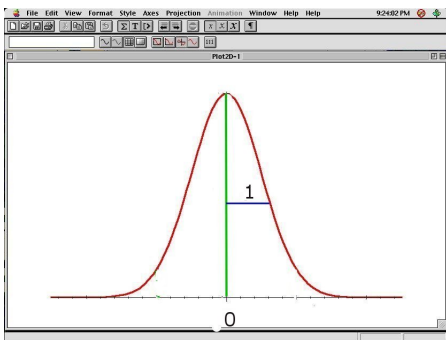
Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

$N(0,1)$



$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

Normal Distribution Formula

Math 1710
Class 11

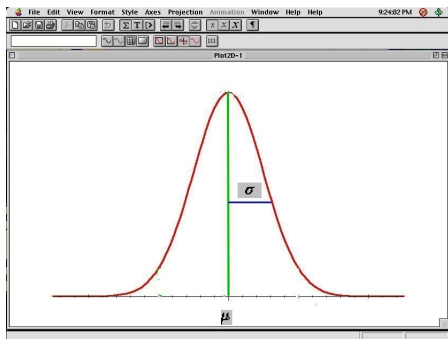
V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation



$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Normal Distribution Formula

Math 1710
Class 11

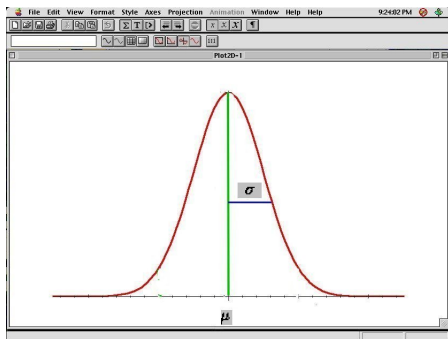
V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation



$N(\mu, \sigma)$

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

These formulas and the following argument are far above the basic level of our course.

Why Normal Out of Binomial?

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

Suppose $X \sim \text{Binom}(2m, .5)$

Why Normal Out of Binomial?

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

Suppose $X \sim \text{Binom}(2m, .5)$
 $\mu = m$ and $\sigma = \sqrt{.5m}$.

Why Normal Out of Binomial?

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

Suppose $X \sim \text{Binom}(2m, .5)$

We want to understand why $X \sim N(m, \sqrt{.5m})$ approximately.

Why Normal Out of Binomial?

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

Suppose $X \sim \text{Binom}(2m, .5)$

We want to understand why $X \sim N(m, \sqrt{.5m})$ approximately.

Set

$$a_k = P(X = m + k) = \binom{2m}{m+k} (.5)^{2m}$$

Why Normal Out of Binomial?

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

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Set

$$a_k = P(X = m + k) = \binom{2m}{m+k} (.5)^{2m}$$

The z-score of $m + k$ is $\frac{k\sqrt{2}}{\sqrt{m}}$.

Why Normal Out of Binomial?

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

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Set

$$a_k = P(X = m + k) = \binom{2m}{m+k} (.5)^{2m}$$

The z-score of $m + k$ is $\frac{k\sqrt{2}}{\sqrt{m}}$.

So we want to show $a_k \sim ce^{-\frac{k^2}{m}}$.

Why Normal Out of Binomial?

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

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Normal
Approximation

Suppose $X \sim \text{Binom}(2m, .5)$

We want to understand why $X \sim N(m, \sqrt{.5m})$ approximately.

Set

$$a_k = P(X = m + k) = \binom{2m}{m+k} (.5)^{2m}$$

So we want to show $a_k \sim ce^{-\frac{k^2}{m}}$.

i.e. $\ln a_k \sim \ln c - \frac{k^2}{m}$.

Why Normal Out of Binomial?

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

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i.e. $\ln a_k \sim \ln c - \frac{k^2}{m}$.

Strategy: Compare a_k to a_0 using the approximation
 $\ln(1+x) \sim x$ for x small.

Why Normal Out of Binomial?

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

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We want to understand why $X \sim N(m, \sqrt{.5m})$ approximately.

Set

$$a_k = P(X = m + k) = \binom{2m}{m+k} (.5)^{2m}$$

i.e. $\ln a_k \sim \ln c - \frac{k^2}{m}$.

$$\begin{aligned} a_k &= \frac{(2m)! (.5)^{2m}}{(m+k)!(m-k)!} = a_0 \frac{(m)(m-1)\dots(m-k+1)}{(m+k)(m+k-1)\dots(m+1)} \\ &= a_0 \frac{(1)(1-\frac{1}{m})\dots(1-\frac{k-1}{m})}{(1+\frac{k}{m})(1+\frac{k-1}{m})\dots(1+\frac{1}{m})} \end{aligned}$$

Why Normal Out of Binomial?

Suppose $X \sim \text{Binom}(2m, .5)$

We want to understand why $X \sim N(m, \sqrt{.5m})$ approximately.

Set

$$a_k = P(X = m + k) = \binom{2m}{m+k} (.5)^{2m}$$

i.e. $\ln a_k \sim \ln c - \frac{k^2}{m}$.

$$\begin{aligned} a_k &= \frac{(2m)! (.5)^{2m}}{(m+k)!(m-k)!} = a_0 \frac{(m)(m-1)\dots(m-k+1)}{(m+k)(m+k-1)\dots(m+1)} \\ &= a_0 \frac{(1)(1-\frac{1}{m})\dots(1-\frac{k-1}{m})}{(1+\frac{k}{m})(1+\frac{k-1}{m})\dots(1+\frac{1}{m})} \end{aligned}$$

So using $\ln(1+x) \sim x$,

$$\ln a_k \sim \ln a_0 - 2 \left(\frac{1}{m} + \frac{2}{m} + \dots + \frac{k-1}{m} \right) - \frac{k}{m}$$

Why Normal Out of Binomial?

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

Suppose $X \sim \text{Binom}(2m, .5)$

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i.e. $\ln a_k \sim \ln c - \frac{k^2}{m}$.

So using $\ln(1+x) \sim x$,

$$\ln a_k \sim \ln a_0 - 2 \left(\frac{1}{m} + \frac{2}{m} + \dots + \frac{k-1}{m} \right) - \frac{k}{m}$$

But $1 + 2 + \dots + k - 1 = \frac{k(k-1)}{2}$, so

Why Normal Out of Binomial?

Suppose $X \sim \text{Binom}(2m, .5)$

We want to understand why $X \sim N(m, \sqrt{.5m})$ approximately.

Set

$$a_k = P(X = m + k) = \binom{2m}{m+k} (.5)^{2m}$$

i.e. $\ln a_k \sim \ln c - \frac{k^2}{m}$.

So using $\ln(1+x) \sim x$,

$$\ln a_k \sim \ln a_0 - 2 \left(\frac{1}{m} + \frac{2}{m} + \dots + \frac{k-1}{m} \right) - \frac{k}{m}$$

But $1 + 2 + \dots + k - 1 = \frac{k(k-1)}{2}$, so

$$\ln a_k \sim \ln a_0 - \frac{k^2}{m}$$

as desired.

C

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation

C

Math 1710
Class 11

V2c

Last Time

More
Complicated
Chapters 6, 17

A Random
Variable
Problem

Proof of
Normal
Approximation