Math 1710 Class 6

Random Variable Operations
Dr. Back

Sep. 9, 2009

Announcements

Please bring your calculator to class on Friday.

We'll discuss using it for binomial and normal calculations.

Means are very intuitive!

$$\mu = \Sigma p_i x_i$$

- $\mu_{X+Y} = \mu_X + \mu_Y$
- $\mu_{X+c} = \mu_X + c$
- $\mu_{cX} = c\mu_X$

Std. Dev. for cX and $X + c$ are pretty natural.

$$\mathrm{Var}(X) = \Sigma p_i (x_i - \mu)^2$$

- $\mathrm{Var}(X + c) = \mathrm{Var}(X)$.
- $\sigma_{X+c} = \sigma_X$
- $\mathrm{Var}(cX) = c^2 \mathrm{Var}(X)$.
- $\sigma_{cX} = |c| \sigma_X$. So $\boldsymbol{\sigma_X = \sigma_{-X}}$.

Variance of a Sum of *Independent* RV's

If X and Y are independent RV's:

$$\mathrm{Var}(X+Y) = \mathrm{Var}(X) + \mathrm{Var}(Y)$$

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e.g. U has standard deviation $\sqrt{2.917} = 1.708$.

U	**probability**
1	$\frac{1}{6}$
2	$\frac{1}{6}$
3	$\frac{1}{6}$
4	$\frac{1}{6}$
5	$\frac{1}{6}$
6	$\frac{1}{6}$

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So $T = U_1 + U_2$ has variance ?

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So $T = U_1 + U_2$ has variance ?
$Var(T) = 2(2.917)$ and

$$\sigma_T = \sqrt{5.834} = 2.415.$$

Y a Binomial(n,p) RV

Y is the number of heads in n tosses of a coin, p being the probability of a head.

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were X_i is a Bernoulli(p) RV describing the i'th toss.
So $\mu_Y = p + p + \ldots + p = np$.
And $\mathbf{Var}(Y) = pq + pq + \ldots + pq = npq$.
Making $\sigma_Y = \sqrt{npq}$.

X and Y indep. RV's

	Mean	**Std Dev**
X	120	12
Y	300	16

μ, σ for $.8Y$?

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$$E(.8Y) = .8E(Y) = .8(300) = 240.$$

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$$Var(.8Y) = ?$$

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$$Var(.8Y) = .8^2 Var(Y)$$

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μ, σ for $.8Y$?

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$$Var(.8Y) = .8^2 Var(Y) = .8^2 \cdot 16^2 = 163.84$$

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μ, σ for $.8Y$?

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$$Var(.8Y) = .8^2 Var(Y) = .8^2 \cdot 16^2 = 163.84$$

$$\sigma_{.8Y} = \sqrt{163.84} = 12.8$$

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$\sigma_{.8Y} = \sqrt{163.84} = 12.8$
$\sigma_{.8Y} = .8 \cdot 16 = 12.8$ is also ok.

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μ, σ for $2X - 100$?

$$E(2X - 100) = 2E(X) - 100 = 2(120) - 100 = 140.$$

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$$Var(2X - 100) = 2^2 Var(Y)$$

X and Y indep. RV's

	Mean	**Std Dev**
X	120	12
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μ, σ for $2X - 100$?

$$E(2X - 100) = 2E(X) - 100 = 2(120) - 100 = 140.$$

$$Var(2X - 100) = 2^2 Var(X) = 2^2 \cdot 12^2 = 576$$

X and Y indep. RV's

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$\sigma_{2X-100} = 2 \cdot 12 = 24$ is also ok.

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μ, σ for $3X - Y$?

X and Y indep. RV's

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μ, σ for $3X - Y$?

$$E(3X - Y) = 3EX - EY = 3(120) - 300 = 60.$$

X and Y indep. RV's

	Mean	**Std Dev**
X	120	12
Y	300	16

μ, σ for $3X - Y$?

$$E(3X - Y) = 3EX - EY = 3(120) - 300 = 60.$$

$$Var(3X - Y) = ?$$

X and Y indep. RV's

	Mean	**Std Dev**
X	120	12
Y	300	16

μ, σ for $3X - Y$?

$$E(3X - Y) = 3EX - EY = 3(120) - 300 = 60.$$

Using independence,

$$Var(3X - Y) = 3^2 Var(X) + Var(Y)$$

X and Y indep. RV's

	Mean	**Std Dev**
X	120	12
Y	300	16

μ, σ for $3X - Y$?

$$E(3X - Y) = 3EX - EY = 3(120) - 300 = 60.$$

$$Var(3X - Y) = 3^2 Var(X) + Var(Y) = 3^2 \cdot 12^2 + 16^2 = 1552$$

X and Y indep. RV's

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X	120	12
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μ, σ for $3X - Y$?

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$$Var(3X - Y) = 3^2 Var(X) + Var(Y) = 3^2 \cdot 12^2 + 16^2 = 1552$$

$$\sigma_{3X-Y} = \sqrt{1552} = 39.395$$

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$\sigma_{3X-Y} = \sqrt{1552} = 39.395$

Here computing the variance and being systematic helped!

X and Y indep. RV's

	Mean	**Std Dev**
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μ, σ for $Y_1 + Y_2$?

X and Y indep. RV's

	Mean	**Std Dev**
X	120	12
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μ, σ for $Y_1 + Y_2$?

(Y_1, Y_2 means two independent copies of Y.)

X and Y indep. RV's

	Mean	**Std Dev**
X	120	12
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μ, σ for $Y_1 + Y_2$?

$$E(Y_1 + Y_2) = E(Y_1) + E(Y_2) = 300 + 300 = 600.$$

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μ, σ for $Y_1 + Y_2$?

$$E(Y_1 + Y_2) = E(Y_1) + E(Y_2) = 300 + 300 = 600.$$

$$Var(Y_1 + Y_2) = ?$$

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	Mean	**Std Dev**
X	120	12
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μ, σ for $Y_1 + Y_2$?

$$E(Y_1 + Y_2) = E(Y_1) + E(Y_2) = 300 + 300 = 600.$$

Using independence,

$$Var(Y_1 + Y_2) = Var(Y_1) + Var(Y_2) = 2Var(Y)$$

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	Mean	**Std Dev**
X	120	12
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μ, σ for $Y_1 + Y_2$?

$$E(Y_1 + Y_2) = E(Y_1) + E(Y_2) = 300 + 300 = 600.$$

$$Var(Y_1 + Y_2) = 2Var(Y) = 2 \cdot 16^2 = 512.$$

X and Y indep. RV's

	Mean	**Std Dev**
X	120	12
Y	300	16

μ, σ for $Y_1 + Y_2$?

$$E(Y_1 + Y_2) = E(Y_1) + E(Y_2) = 300 + 300 = 600.$$

$$Var(Y_1 + Y_2) = 2Var(Y) = 2 \cdot 16^2 = 512.$$

$$\sigma_{Y_1+Y_2} = \sqrt{512} = 22.63.$$

$X_1 + X_2$ vs. $2X$

Tossing one fair coin is described by Bernoulli(.5):

X	**probability**
0	.5
1	.5

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The RV $2X$?

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The RV $2X$:

$2X$	**probability**
0	.5
2	.5

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The RV $2X$:

$2X$	**probability**
0	.5
2	.5

If X_1 and X_2 are independent copies of X, then $X_1 + X_2$ can come out to 0,1, or 2.

$X_1 + X_2$ vs. $2X$

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0	.5
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The RV $2X$:

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0	.5
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The RV $2X$:

$2X$	**probability**
0	.5
2	.5

The RV $X_1 + X_2$:

$X_1 + X_2$	**probability**
0	.25
1	.5
2	.25

How many heads in total?

How can these two possibilities come up in tossing 2 coins?

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Method 1: Just toss them.

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Method 2: Toss one coin.
Then turn the other coin over to the same result.

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Method 1: Just toss them.

This is $X_1 + X_2$.

Method 2: Toss one coin.
Then turn the other coin over to the same result.

Note the second coin is still random.

How many heads in total?

How can these two possibilities come up in tossing 2 coins?

Method 1: Just toss them.

This is $X_1 + X_2$.

Method 2: Toss one coin.
Then turn the other coin over to the same result.

This is $2X$.

Experimentally $X_1 + X_2$ and 2X

$X_1 + X_2$	**frequency**
0	?
1	?
2	?

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0	?
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$\bar{x}, s$?

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$X_1 + X_2$	**frequency**
0	?
1	?
2	?

These $\bar{x}$,s should be close to $\mu = 1$, $\sigma = \sqrt{2(.5)(.5)} = .707$ resp.

Experimentally $X_1 + X_2$ and 2X

$2X$	**frequency**
0	?
2	?

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7 Balls Randomly into 5 boxes

What is the probability that the first box remains empty?

What is the expected number of empty boxes?

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Let X_i be an RV which is 1 if box i is empty, 0 otherwise.

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What is the probability that the first box remains empty?

What is the expected number of empty boxes?

The 1st question is a great hint for easily doing the 2nd!

Let X_i be an RV which is 1 if box i is empty, 0 otherwise.

What does $Y = X_1 + X_2 + X_3 + X_4 + X_5$ represent?

Ch 15 #41. Sobriety checkpoints.

Officers ask questions, then maybe detain for a breathylyzer test.

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Question a) in terms of this notation?

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$P(B+|not\ DR)$

Ch 15 #41. Sobriety checkpoints.

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Notation:

DR= someone has been drinking.

$B+=$ someone is held for breathylyzer

$B-=$ someone is not held for breathylyzer

$P(B+|not\ DR) = 1 - P(B-|not\ DR) = 1 - .8 = .2$

(Essentially given.)

Ch 15 #41. Sobriety checkpoints.

b) P(being detained)?

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b) P(being detained)?
i.e. P(B+)?

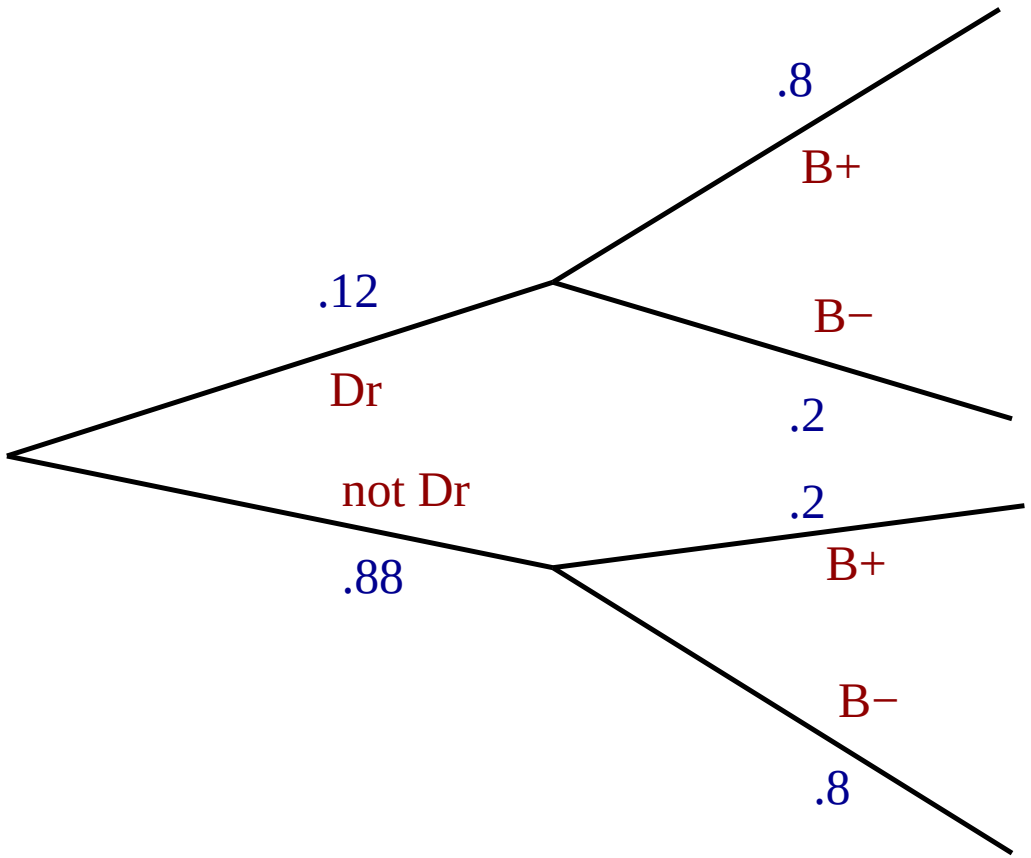
Ch 15 #41. Sobriety checkpoints.

i.e. P(B+)?
Given:

$$P(DR) = .12$$
$$P(B+|DR) = .8$$
$$P(B-|not\ DR) = .8.$$

Ch 15 #41. Sobriety checkpoints.

i.e. P(B+)?



Ch 15 #41. Sobriety checkpoints.

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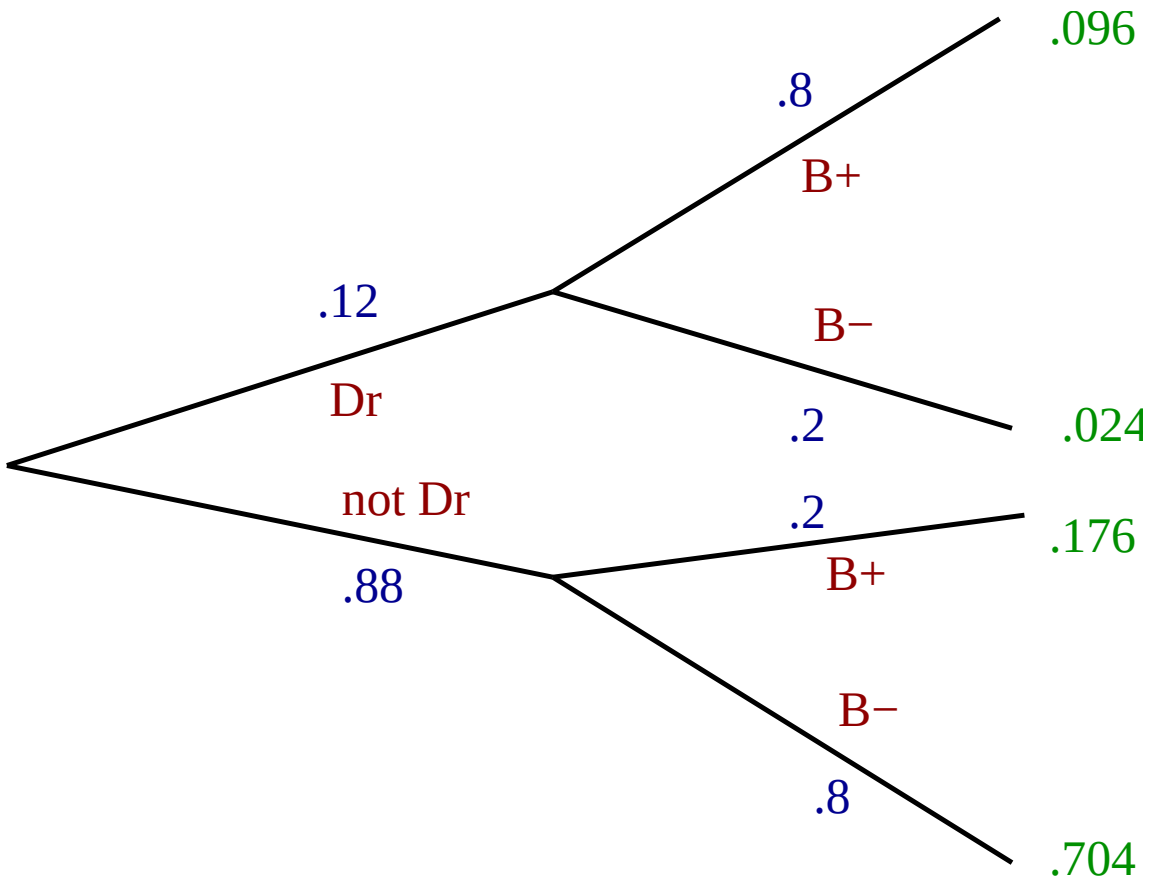

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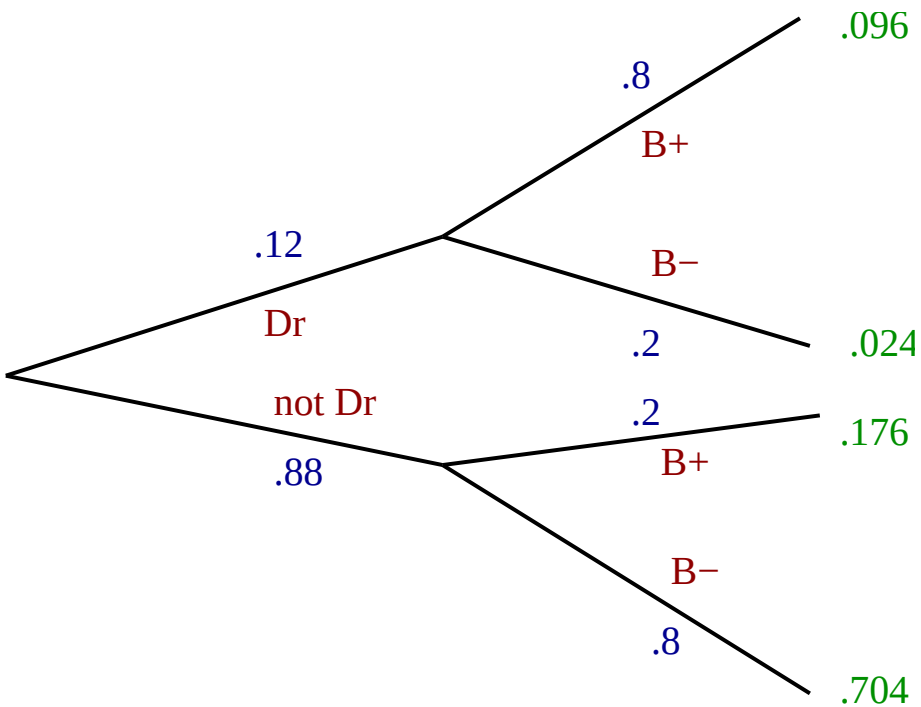

$P(B+) = .096 + .176 = .272$ (from tree)

Ch 15 #41. Sobriety checkpoints.

c) P(someone detained has been drinking)?

Ch 15 #41. Sobriety checkpoints.

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i.e. $P(DR|B+)$?

Ch 15 #41. Sobriety checkpoints.

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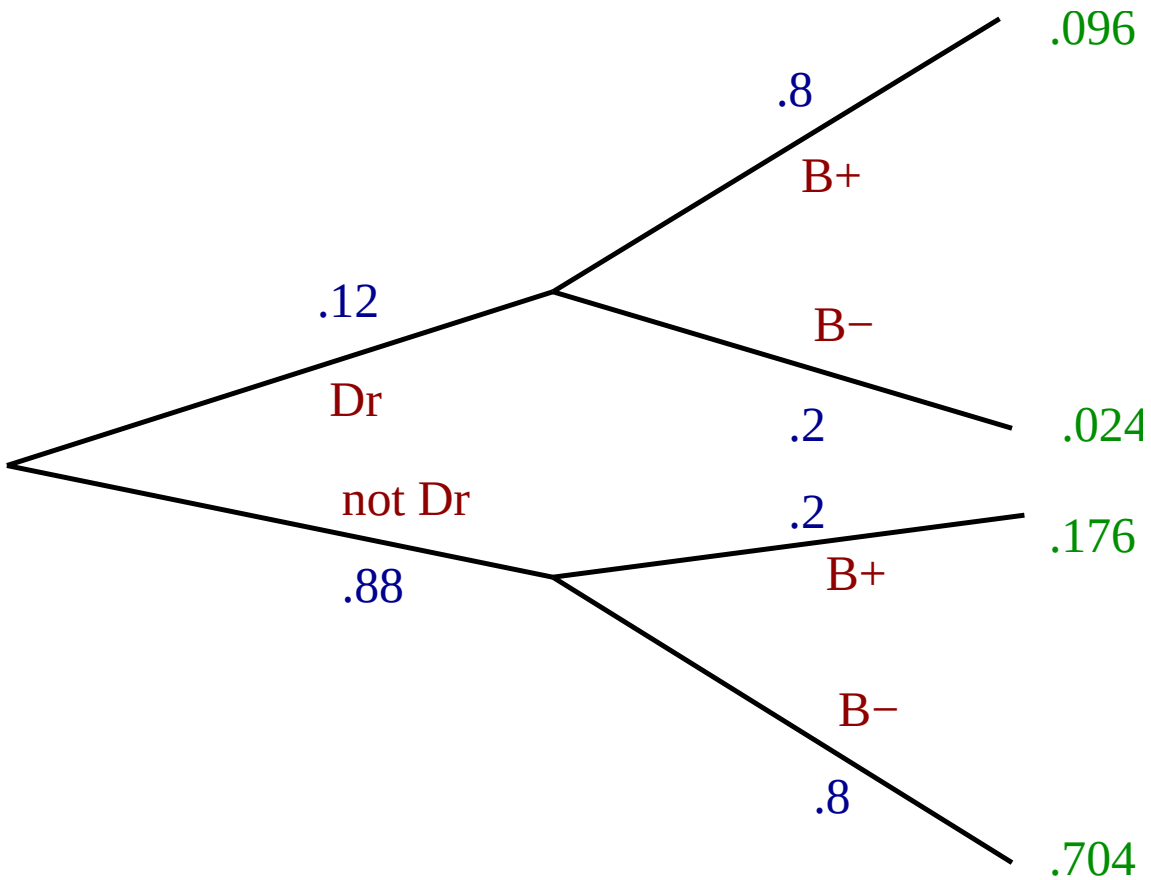
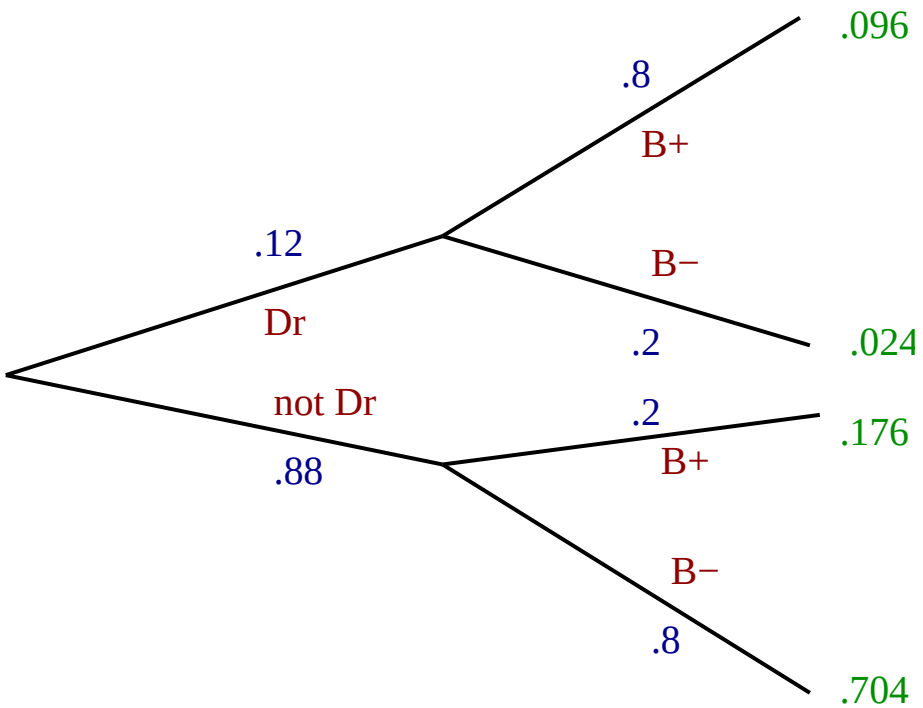


Ch 15 #41. Sobriety checkpoints.

i.e. $P(DR|B+)$?



$$P(DR|B+) = \frac{P(DR\&B+)}{P(B+)} = \frac{.096}{.272} = .353.$$

(from tree)

Ch 15 #41. Sobriety checkpoints.

d) P(someone released has been drinking)?

Ch 15 #41. Sobriety checkpoints.

d) P(someone released has been drinking)?
i.e. $P(DR|B-)$?

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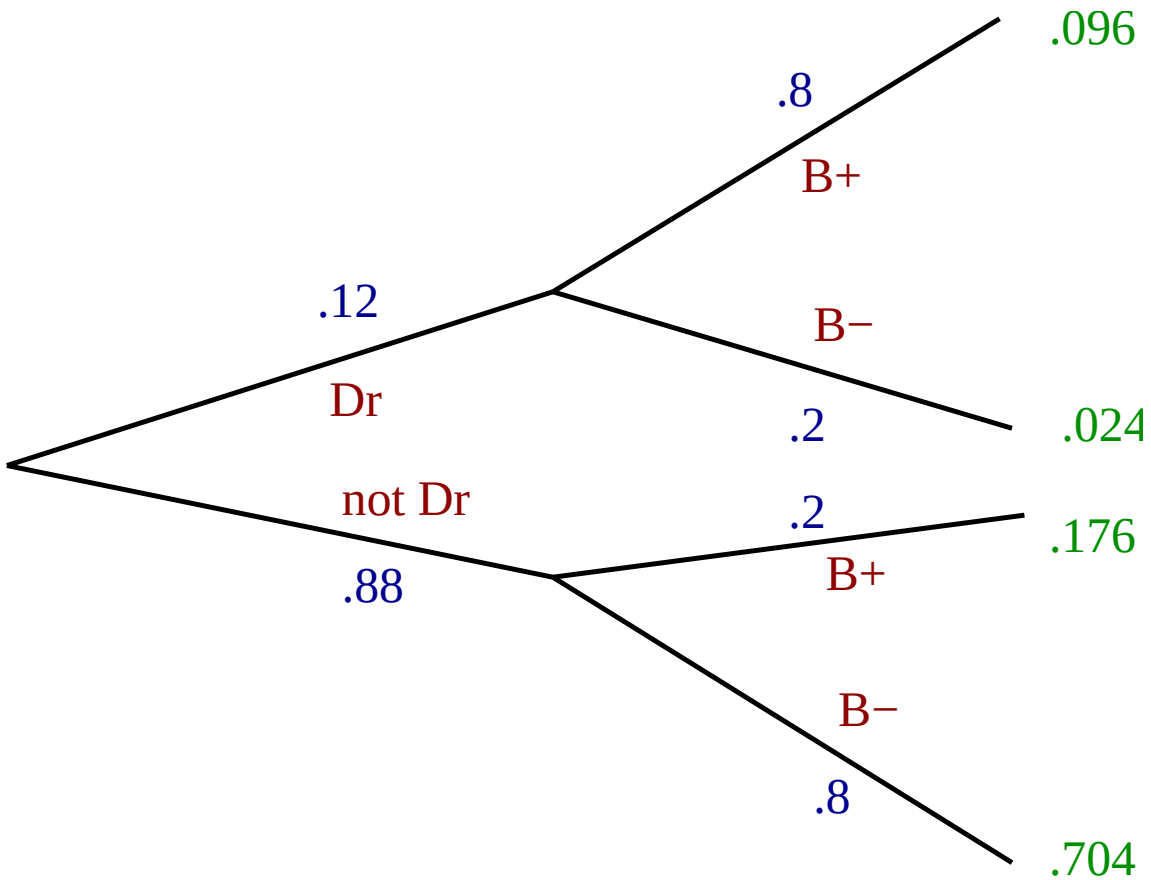
i.e. $P(DR|B-)$?



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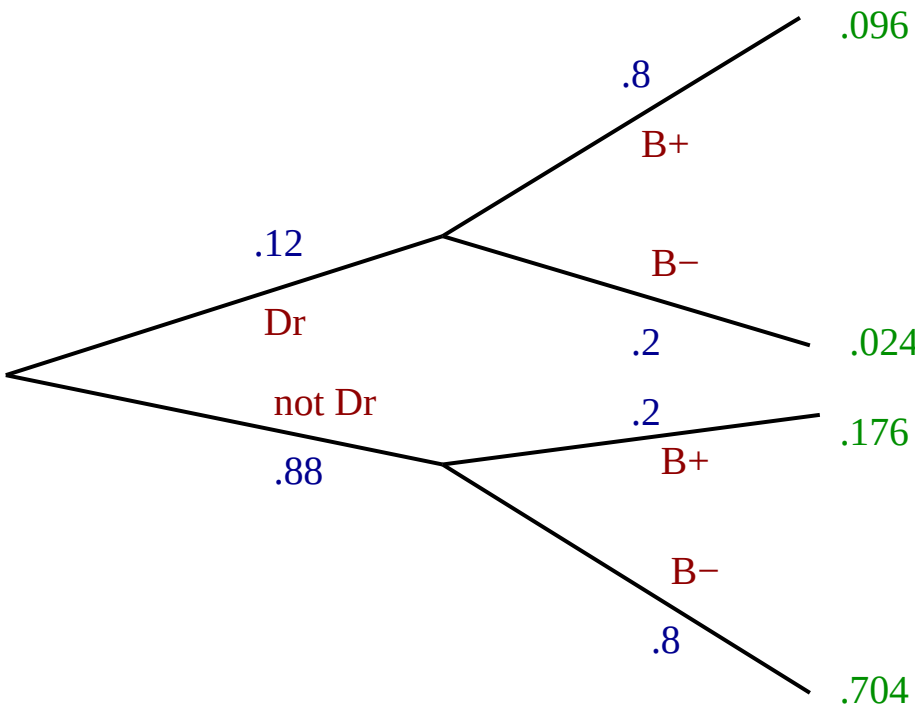
i.e. $P(DR|B-)$?



$$P(DR|B+) = \frac{P(DR\&B-)}{P(B-)} = \frac{.024}{.728} = .033.$$

(from tree)