Why Normal Out of Binomial?

Suppose $X \sim \text{Binom}(2m, .5)$.

$\mu = m \qquad \sigma = \sqrt{.5m}$

So why is $X \sim N(m, \sqrt{.5m})$ approximately?

Set $a_k = P(X = m+k) = \binom{2m}{m+k}(.5)^{2m}$

The Z score of $m+k$ is $k\sqrt{2}/\sqrt{m}$.

So we want to show $a_k \sim ce^{-k^2/m}$

i.e. $\ln a_k \sim \ln c - \frac{k^2}{m}$

Strategy: Compare a_k to a_0 using the approximation

$$\ln(1+x) \approx +x$$

For x small.