Smith Theory by Example

Allen Back

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Sometimes the universal coefficients thm lets us go from these cases to coefficient ring Z.

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Smith (Annals of Math, 1938): Fixed point set of a Z_p action on a homology n-sphere is a Z_p homology r-sphere with $r \leq n$.
Smith did the Q case for toral actions too.

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Borel: (Anls Math Stds Smnr on Transformation Groups $\sim'$ 60) Introduced the Borel construction (homotopy quotient) in the hope of doing Smith theory more systematically.

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Progress still took a while.

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Wu-Yi Hsiang Reformulation: ($\sim'$ 70) Organized around the philosophy that (for the above coeff. in the above grp cases) the free part of the equivariant cohomology goes most of the way towards determining the cohomology of the fixed point set. The torsion goes towards determining the orbit structure.

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For non-abelian Lie groups G this doesn't work. The Hsiangs already showed given any finite complex K, there is a compact fintie dimensional acylic G-space X on which G acts with fixed point set $X^G = K$,

Lowell Jones (Annals of Math '71) and Bob Oliver's work (Comm. Math. Helv '75) solidified this idea further.

References

[CTTTG] Cohomology Theory of Topological Transformation Groups, Wu-Yi Hsiang, 1975.

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For example the 17 pages of Chapter 2 are an almost complete development of the structure, classification, and rep theory of compact Lie groups.

References

[CTTTG] Cohomology Theory of Topological Transformation Groups, Wu-Yi Hsiang, 1975.
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[CMTG] Cohomological Methods in Transformation Groups, Chris Allday and Volker Puppe 1993.

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The bible, but sometimes a little terse.
[CMTG] Cohomological Methods in Transformation Groups, Chris Allday and Volker Puppe 1993.
(Cambridge Studies in Advanced Mathematics 32)
A nice recent book.
Chris was Wu-Yi's first student after moving to Berkeley.

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[CTTTG] Cohomology Theory of Topological Transformation Groups, Wu-Yi Hsiang, 1975.
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The bible, but sometimes a little terse.
[CMTG] Cohomological Methods in Transformation Groups, Chris Allday and Volker Puppe 1993.
[Brd] Introduction to Compact Transformation Groups, Glen Bredon 1972.

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[CMTG] Cohomological Methods in Transformation Groups, Chris Allday and Volker Puppe 1993.
[Brd] Introduction to Compact Transformation Groups, Glen Bredon 1972.
Chapter 3 is a bare-handed approach to Smith theory.

Weights as Cohomology Classes

Unless otherwise specified, the coefficient ring k is Q in the case of toral $G = ((S^1)^n)$ actions and Z_p in the case of $G = Z_p$ tori $(\oplus(Z_p))$ actions.

Weights as Cohomology Classes

$BS^1 = CP^\infty$, $B(Z_2) = RP^\infty$.
So for G a torus or Z_2-tori,

$$H^*(BG) = k[t_1, \ldots, t_r]$$

where deg t_i is 2 or 1 respectively.

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For p odd,

$$H^*(BG) = k[t_1, \ldots, t_r] \otimes \wedge[\nu_1, \ldots, \nu_r]$$

where $deg(t_i) = 2$ and $deg(\nu_i) = 1$.

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In all these toral cases, for a (possibly p) torus of rank r, we use the notation

$$R := k[t_1, \ldots, t_r]$$

for the polynomial part of the coefficient ring of equivariant cohomology H^*_G and R_0 for its field of fractions.

Weights as Cohomology Classes

A homomorphism $H \to G$ induces a map $BH \to BG$ and so a map $H^*(BG) \to H^*(BH)$.

Weights as Cohomology Classes

Thus a weight of a linear representation as a representation $w : G \to S^1$ is equally well viewed as a 2 dimensional cohomology class representing the pullback to BG of the generator in dim 2 of $H^*(CP^\infty)$. Also interpretable via transgression in the spectral sequence for $G \to EG \to BG$. Similarly in the p odd case of Z_p, weights are 2 dimensional cohomology classes and in the Z_2 case 1 dimensional cohomology classes.

Wu-Yi's Reformulation [CTTTG page 47]

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is the polynomial part of $H^*_G(pt) = H^*(BG)$ and

R_0 is its field of fractions.

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R_0 is its field of fractions.

Let $\{\xi_1, \ldots, \xi_l, \nu_1, \ldots, \nu_m\}$ be a generator system for the R_0 algebra

$$H^*_G(X, k) \otimes_R R_0$$

with the ξ_i of even degree and and the ν_j of odd degree.

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$F = F^1 + \ldots + F^s$, F_j connected, $q_j \in F^j$

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$F = F^1 + \ldots + F^s$, F_j connected, $q_j \in F^j$
$I := ker\rho : A = R_0[t_1, \ldots, t_l] \otimes \wedge_{R_0}[\nu_1, \ldots, \nu_m] \rightarrow H^*_G(X, k) \otimes R_0$.

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Then the radical of I decomposes

$$\sqrt{I} = M_1 \cap M_2 \cap \ldots \cap M_s$$

where the M_j are maximal ideals $M(\alpha_j)$
with $\alpha_j = (\alpha_{j1}, \ldots, \alpha_{jl}) \in R_0^l$.

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Components F^j of the fixed point set are in 1:1 correspondence with the α_j so that $\iota : q_j \in F^j \to X$ satisfies $\iota_j^* \xi_i = \alpha_{ji}$.

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$$H^*(F^j, k) \otimes_k R_0 \cong A/I_j$$

where $I_j = I_{M_j} \cap A$ and I_{M_j} is the localization of I at M_j.

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$I = I_1 \cap I_2 \cap \ldots \cap I_s = I_1 \cdot I_2 \cdot \ldots \cdot I_s$

Borel Criterion

View $X \to X_G \to BG$ more explicitly as $X \to X \times_G EG \to G \backslash EG$ where elements of X_G are $[x, e]$ with $x \in X$, $e \in EG$, and $[x, e] \sim [xg, g^{-1}e]$.

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If the fixed point set is nonempty (say $q \in F$), then the map $Ge \to [q, g]$ gives a section of $X \to X \times_G EG \to G \backslash EG$.

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View elements of BG as cosets Ge with $e \in EG$.
If the fixed point set is nonempty (say $q \in F$), then the map
$Ge \to [q, g]$ gives a section of $X \to X \times_G EG \to G \backslash EG$.
Borel Criterion: The fixed point set F is nonempty iff
$H^*(BG) \to H^*(X_G)$ is injective.

Acyclic Manifolds

Serre spectral sequence applied to $X \to X_G \to BG$ shows

$$H_G^*(X) = R.$$

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$H_G^*(X) = R$ and $H^*(F^j, k) \otimes_k R_0 \cong R/I_{M_j}$ means F^j must be acyclic.

Sphere

Sps. X is a cohomology sphere with k coefficients. The E_2 term of the spectral sequence for $X \to X_G \to BG$ is then $R \otimes H^*(X)$.

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Because the spectral sequence has only 2 rows, there is only one possible nonzero differential. And $H^*(BG) \to H^*(X_G)$ is injective iff the spectral sequence collapses.

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Because the spectral sequence has only 2 rows, there is only one possible nonzero differential. And $H^*(BG) \to H^*(X_G)$ is injective iff the spectral sequence collapses.
So by the Borel criterion, there is a nonempty fixed point set exactly when the spectral sequence collapses at the E_2 stage.

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When $F \neq \phi$,

$$H^*_G(X) \otimes_R R_0 \cong R_0[x]/(f(x))$$

where $f(x) = x^2 + ax + b$ is the quadratic polynomial expressing x^2 in terms of our module basis $\{1, x\}$ of $H^*_G(X)$ as an R-module.

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$$H_G^*(X) \otimes_R R_0 \cong R_0[x]/(f(x))$$

By the rationality part of Wu-Yi's fundamental fixed point theorem, $f(x) = (x - \alpha_1)(x - \alpha_2)$ in $R_0[x]$.

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Localizing at the maximal ideals tells us F is two points (S^0) if $\alpha_1 \neq \alpha_2$ and S^r if $\alpha_1 = \alpha_2$

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This is the original Smith theorem.

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Note it is really the multiplicative structure of the cohomology ring which distinguishes examples.

Products of Spheres

Projective Spaces

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$$H^*_G(X) \cong H^*(X) \otimes_k R$$

as a module. (Also the Leray-Hirsch thm)

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Let x be the lift of a generator of $H^2(X)$ to $H^2(X_G)$. Then as a ring

$$H^*(X_G) = R[x]/(f(x))$$

where $f(x)$ is a monic polynomial of degree n.

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For parity reasons, the spectral sequence of $X \to X_G \to BG$ collapses and

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as a module. (Also the Leray-Hirsch thm)
By the rationality part of Wu-Yi's fundamental fixed point theorem,

$$H^*(X_G) \otimes_R R_0 = R_0[x]/(f(x))$$

where $f(x) = \Pi(x - \alpha_i)$ in $R_0[x]$.

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For parity reasons, the spectral sequence of $X \to X_G \to BG$ collapses and

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as a module. (Also the Leray-Hirsch thm)
By the Gauss lemma, $f(x)$ already splits into linear factors over R.

$$f(x) = \Pi_i = 1^s(x - \alpha_i)_i^m$$

Projective Spaces

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For parity reasons, the spectral sequence of $X \to X_G \to BG$ collapses and

$$H^*_G(X) \cong H^*(X) \otimes_k R$$

as a module. (Also the Leray-Hirsch thm)
Localization at the maximal ideals then gives s fixed point components, each a cohomology CP^{m_i-1}.

Involutions on Grassmannian of 2-planes

My Ph. D. thesis.
Was once accepted "subject to revision" for the Memoirs but the editor wanted a less computational proof which I have never known how to do.

Involutions on Grassmannian of 2-planes

Using the splitting principle, view

$$H^*(G_2(R^N), Z_2) \subset Z_2[t_1, t_2].$$

where the t_i are of degree 1.

Involutions on Grassmannian of 2-planes

Let h_i be the i'th complete symmetric function of t_1, t_2. So

$$h_k = s_{k+1}/s_1$$

where s_k is the k'th power sum, $t_1^k + t_2^k$ in this case.

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The h_k are the Stiefel-Whitney classes of the canonical $N-1$ plane bundle over $G_2(R^N)$.

Involutions on Grassmannian of 2-planes

Prop:

- $H^*(G_2(R^N), Z_2) = Z_2[h_1, h_2]/(h_{N-1}, h_N)$.
- $\{h_a h_b : 0 \leq a \leq N-2, 0 \leq b \leq N-2\}$ form a module basis for this ring.
- $$h_a h_b h_c = -\Sigma_{j=0}^{a-1} h_j h_{a+b+c-j} + \Sigma_{j=0}^{a} h_{b+j} h_{a+c-j}$$

  even over Z.

Involutions on Grassmannian of 2-planes

Linear models: Linear representations of $G = Z_2$ into $Gl(N)$ are indexed by the number l of $-1's$. The two canonical bundles over $X = G_2(R^N)$ are equivariant with respect to such an action, so for a linear action, the spectral sequence of $X \to X_G \to BG$ collapses. Hence for a linear action

$$H^*(X_G) = R[h_1, h_2]/(f^{N-1}, f^N)$$

and it is not hard to show that

$$f^{N-k} = \Sigma_{j=0}^{N-k} \rho_j h_{N-k-j}$$

where ρ_j is the j'th elementary symmetric function of the weights (counting multiplicities) of the Z_2 representation.

Involutions on Grassmannian of 2-planes

Based on Wu-Yi's version of the fixed point theorem:
Prop: If an involution has the same equivariant cohomology as a linear model, then cohomologically its fixed point sets are also the same.

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Prop: If an involution has the same equivariant cohomology as a linear model, then cohomologically its fixed point sets are also the same.

In general we have three components, $G_2(R^l)$, $RP^{l-1} \times RP^{N-l-1}$, and $G_2(R^{N-l})$.

Involutions on Grassmannian of 2-planes

For a general involution, consider the Serre spectral sequence for $X \to X_G \to BG$.

Prop:

- If N is even and not congruent to 64 mod 192, then the action of $\pi_1(BG)$ on $H^*(X, Z_2)$ is trivial.
- For N even, if there is a nonempty fixed point set, then the spectral sequence collapses at the E_2 level.

Involutions on Grassmannian of 2-planes

Thus under the above dimension restrictions, for any involution, if $F \neq \phi$, we have fairly quickly shown that

$$H_G^*(X) = Z_2[h_1, h_2, x]/(f, g)$$

where $f = h_{N-1} \pmod{x}$ and $g = h_N \pmod{x}$.

Involutions on Grassmannian of 2-planes

90% of the work in my thesis is using the Steenrod algebra invariance of (f, g) to show that f and g must be the same polynomials as for one of the linear models.

Involutions on Grassmannian of 2-planes

This gives the theorem that cohomologically (for these values of N), any involution on $G_2(R^N)$ has fixed point set with the same Z_2 cohomology as one of the linear models.

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It is based on showing that a certain mod 2 binomial coefficient is *usually* nonzero.

Unfortunately, that coefficient does sometimes vanish, so there are a lot of special cases that had to be checked to give the full theorem.

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It is based on showing that a certain mod 2 binomial coefficient is *usually* nonzero.

Probably this example is one of the most computationally intricate *examples* of the fixed point theorem that has ever been written down.