MAT344 Problem Set 1
(due Thursday, September 19, 12pm)

Notes:

1. For all the questions, always *explain your reasoning* and refer to the results you are using. Just a number (even if it is the correct final answer) will **not** get you full credit.

2. When submitting to Crowdmark, please ensure that your uploads are *legible, correctly rotated,* and *properly matched with the correct problems.* Any improperly uploaded problem scans will not be graded.

3. Any assignments submitted after the deadline will not be accepted.

Part A

Three of these questions will be marked.

Problem 1. DNA sequences consist of four letters (also called *bases*): A, T, C, and G. The DNA sequences of humans have approximately 4 billion bases (4×10^9). How many possible DNA sequences of length 4 billion are there?

Problem 2. In how many ways can you sit down 8 people around a round table if a family of 4 insists on being seated together? **Hint:** This is trickier than it may seem at first.

Problem 3. In a spaceship factory, there are six spaceship parts that need to be manufactured, but they have to be created one at a time (because there is only one robot capable of doing the job). How many ways are there to manufacture the six parts, if the spaceship engine must be manufactured before the spaceship hull? The other four components can be manufactured at any time.

Problem 4. How many license plates are there of the form $L_1L_2L_3D_1D_2D_3$ where L_1, L_2, L_3 are each capital letters in the English alphabet (of which there are 26), and D_1, D_2, D_3 are each decimal digits (elements of the set
$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$), subject to the restriction that at least one digit is nonzero and at least one letter is K?

Part B

Two of these questions will be marked.

Problem 5. Jorge Luis Borges's "Library of Babel" is made up of hexagonal rooms. Each room has 5 bookshelves on 4 walls, and each shelf contains 32 books. Every book is 410 pages, with 40 lines on each page, and each line contains 80 characters. The characters are spaces, commas, periods, and 22 letters. If every book is unique, and the library contains every possible book, which is larger: the number of rooms in the library, or the largest known prime?

Problem 6. In a tournament of 16 teams, the games proceed in rounds. In the first round, the 16 teams are divided into 8 pairs; the teams within a pair play each other. The winners of those 8 games then proceed to be paired, and they play 4 games. The winners of those proceed to the next round, and are paired. This continues until only one team is left, who is then declared the champion.

1. How many rounds of games are there?

2. How many ways are there of pairing the 16 teams?

3. Given a pairing of the 16 initial teams, how many different outcomes of the first round are there? An outcome of a game is who wins and who loses.

4. How many different possible tournaments are there? Your count must include the number of pairings at each round, as well as the number of different outcomes.

Problem 7. Pizza Pizza offers pizzas with the following options:

1. 4 sizes

2. 6 types of dough

3. 12 sauces (you can only select one of them, one of the 12 options is "no sauce")

4. 4 cheeses (you can only select one of them, one of the 4 options is "no cheese")

5. 19 types of veggie toppings (you may select any number of them)

6. 20 types of meat toppings (you may select any number of them)

7. 6 types of cheese toppings (and unlike with the "cheeses" above, you may select any number of them)

8. 6 types of "other" toppings (you may select any number of them)

Note: Pizza Pizza allows you to order double or triple orders of a single topping, let us just ignore this. In this problem, you are only allowed to either select a topping or not. Also, "no sauce" and "no cheese" are already included in the 12 sauces/4 cheeses, respectively. You will find that there are plenty of options already.

A group of friends eat ten Pizza Pizza pizzas for a meal and they eat 3 meals (breakfast, lunch, and dinner) every day. They never eat the same pizza twice, and they also insist that every pizza they order has

1. exactly 5 meat toppings,

2. exactly 5 veggie toppings,

3. exactly 3 "other" toppings,

and they have been doing this since the beginning of the universe (which you may assume to have been 13.799 billion years ago). Approximately what percentage of options have they exhausted as of today?

Part C

This question will only be marked for completion.

Problem 8. Give an example of a counting problem that can be solved using strings, permutations, or combinations.