MAT344 Problem Set 4
(due Thursday, October 10, noon)

Notes:

1. For all the questions, always *explain your reasoning* and refer to the results you are using. Just a number (even if it is the correct final answer) will **not** get you full credit.

2. When submitting to Crowdmark, please ensure that your uploads are *legible, correctly rotated,* and *properly matched with the correct problems.* Any improperly uploaded problem scans will not be graded.

3. Any assignments submitted after the deadline will not be accepted.

Part A

Three randomly chosen questions from this part will be marked.

Problem 1. Show that given a square of area 1, any way of drawing five dots within the square will yield at least two dots that are within distance $\frac{3}{4}$ of each other.

Problem 2. There are 280 students currently enrolled in MAT344 and six tutorials. You want to make an argument to the U of T administration that current tutorial sizes are too large, but you do not have access to the exact tutorial enrollment statistics. You want to make a statement of the form:

> There is a tutorial with at least n people currently enrolled.

You want to make your statement effective (i.e. n as large as possible), but you also don't want to lie. What is the most effective statement you can make?

Problem 3. Is the graph in Figure 1 Eulerian? If it is, find an Eulerian circuit using the edge labelling. If it is not, explain why it is not.

Problem 4. There are 11 non-isomorphic graphs on 4 vertices. Draw all 11, and under each one indicate: is it connected? Is it a forest? Is it a tree? **Hint:** One has 0 edges, one has 1 edge, two have 2 edges, three have 3 edges, two have 4 edges, one has 5 edges and one has 6 edges.
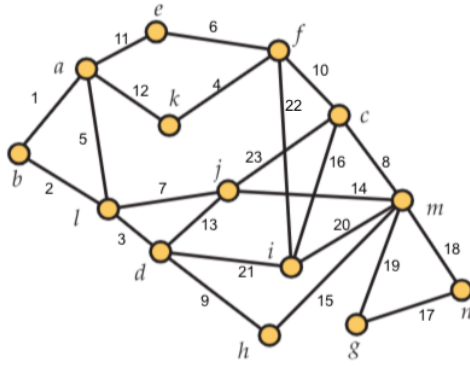
Figure 1: A graph

Part B

Two randomly chosen questions from this part will be marked.

Problem 5. Prove that if you have 100 integers, you can choose 15 of them so that the difference of any two is divisible by 7.

Problem 6. The **complement** $\overline{G}$ of a graph G is the graph with the same vertices as G and xy is an edge of $\overline{G}$ if and only if it is not an edge of G. A graph is **self-complementary** if G is isomorphic to $\overline{G}$. Show that if G is self-complementary then it has $4k$ or $4k+1$ many vertices for some nonnegative integer k. Find a self-complementary graph on 4 vertices and one on 5 vertices

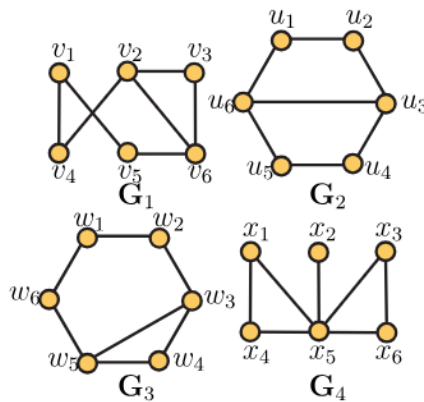**Problem 7.** Consider the four graphs on Figure 2: Determine which (if any) pairs of graphs

Figure 2: Four graphs

are isomorphic. For pairs that are isomorphic, give an isomorphism between the two graphs. For pairs that are not isomorphic, explain why.

Part C

This question will be marked for completion only.

Problem 8. Give an example of two nonisomorphic graphs with the same degree sequence.