MAT344 Problem Set 5
(due Thursday, October 24, noon)

Notes:

1. For all the questions, always *explain your reasoning* and refer to the results you are using. Just a number (even if it is the correct final answer) will **not** get you full credit.
2. When submitting to Crowdmark, please ensure that your uploads are *legible*, *correctly rotated*, and *properly matched with the correct problems*. Any improperly uploaded problem scans will not be graded.
3. Any assignments submitted after the deadline will not be accepted.

Part A

Three randomly chosen questions from this part will be marked.

Problem 1. Is the graph in Figure 1 Hamiltonian? If it is, find an Hamiltonian circuit using the edge labelling. If it is not, explain why it is not.

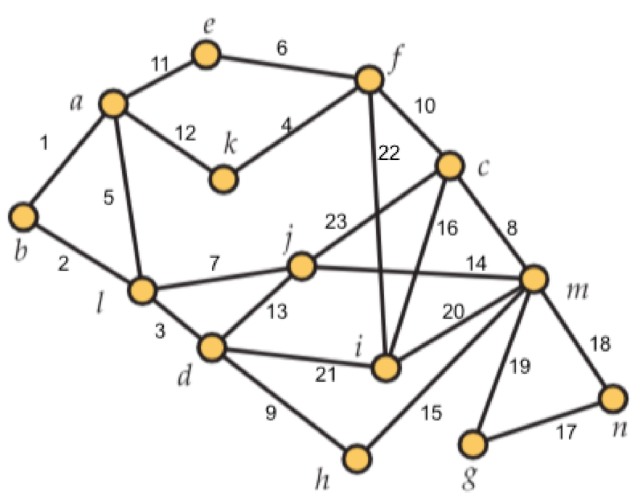
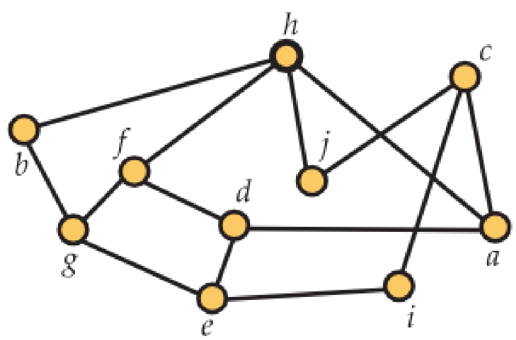


Figure 1: A graph

Problem 2. A pharmaceutical manufacturer is building a new warehouse to store its supply of 10 chemicals it uses in production. However, some of the chemicals cannot be stored in the same room due to undesirable reactions that will occur. The matrix below has a 1 in position (i, j) if and only if chemical i and chemical j cannot be stored in the same room. Model this problem using graph theory and determine the smallest number of rooms into which they can divide their warehouse so that they can safely store all 10 chemicals in the warehouse.

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Problem 3. Find the chromatic number, $\chi(G)$, of the following graph G, and a coloring using $\chi(G)$ colors.



Problem 4. Delete one edge from the complete graph K_5. Find a planar drawing of the resulting graph.

Part B

Problem 5. Recall that the **degree sequence** of a graph is the list of the degrees of each vertex in decreasing order. Consider the following lists of six positive integers:

$$A = [3, 2, 2, 1, 1, 1]$$
$$B = [5, 5, 5, 4, 3, 2]$$
$$C = [5, 5, 4, 4, 3, 3]$$
$$D = [4, 4, 2, 2, 2, 2]$$
$$E = [3, 3, 3, 3, 3, 3]$$

Determine which sequence(s):

1. cannot be the degree sequence of a graph;
2. must be the degree sequence of an eulerian graph;
3. must be the degree sequence of a hamiltonian graph;
4. could be the degree sequence of a tree; and
5. could be the degree sequence of a graph, but cannot be a planar graph.

(As always, explain your answers).

Problem 6. Figure 3 shows the Petersen Graph. Show that the Petersen Graph is nonplanar.

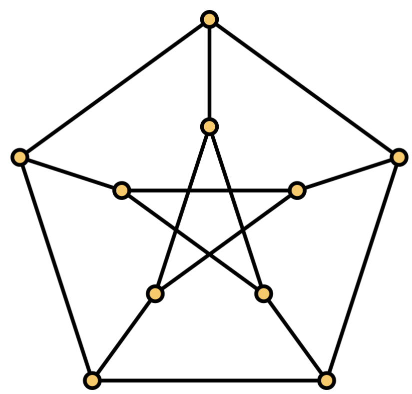


Figure 2: The Petersen Graph

Problem 7. Let G be a connected planar graph with v vertices, e edges and f faces satisfying the following:

- Every vertex has degree $d \geq 3$,
- For any planar drawing, the boundary of every face consists of $b \geq 3$ edges.

(a) Prove that

$$\frac{1}{d} + \frac{1}{b} > \frac{1}{2}$$

(b) For all possible values of (d, b), find v, e, f (you do not need to draw the graphs).

(c) Compare your answer to the classification of Platonic solids. Do you notice any relation to what you found in the previous part?

Part C

This question will be marked for completion only.

Problem 8. Give an example of a real life problem that can be modeled using planar graphs.