Name:________________________________

Final Exam | Math 2310 | Spring 2013

communicate: show work and indicate reasons

open book, open notes, not open people

1) Find the linear combination $\vec{b} = 2\vec{u} + 3\vec{v} + 4\vec{w}$. Then write $\vec{b}$ as a matrix-vector product $A\vec{y}$.

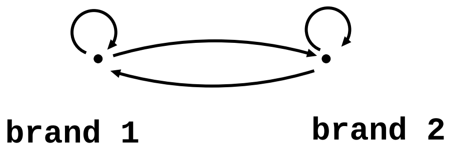
$$\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \quad \vec{w} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}.$$

2) Write an augmented matrix and reduce the system to upper triangular form by two row operations:

$$\begin{aligned} x_1 + 2x_3 &= 8 \\ 2x_1 + x_2 &= 20 \\ 2x_2 + x_3 &= 0 \end{aligned}$$

Solve by back substitution, and *check* to make sure your answer actually solves the equations.

3) Fill in the missing entries in a Markov matrix $A = \begin{bmatrix} .2 & ? \\ ? & .7 \end{bmatrix}$ and mark the diagram with the corresponding probabilities.



Find a steady state.

If initially half of the people use each brand, choose a vector $\vec{x}_0$ to represent this situation, and find the corresponding vectors $\vec{x}_1$ and $\vec{x}_2$ after one and two timesteps.

If the Markov chain predicts the purchasing habits of 1100 people, how many of them will decide to buy brand 1 in the long run?

4) Find an orthogonal matrix which diagonalizes

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

5) Find a function of the form $y = c + \dfrac{d}{t}$ [not a line] which best fits the data $(t, y) = (\frac{1}{4}, 2), (\frac{1}{2}, 1), (1, 1)$.

Sketch a graph of the data and your function.

6) Consider the set of vectors $\vec{x}$ which are perpendicular to both $\vec{y}$ and $\vec{w}$.

$$\vec{y} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \qquad \vec{w} = \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \\ 2 \end{bmatrix}.$$

You are given that this set of vectors $\vec{x}$ is a subspace of one of the Euclidean spaces R^n. Which n is it?

Find a basis and the dimension for this subspace.

Is the vector $\begin{bmatrix} -2 \\ -1 \\ 2 \\ 1 \\ 2 \end{bmatrix}$ in this subspace?