Mathematics 2310: Linear Algebra with Applications

Spring 2012

Preliminary Exam 1 : February 22
This exam is closed book: no notes, calculators or other aids allowed.

1. Set A, B and C to be the matrices

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{pmatrix}, \qquad C = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

   Compute the matrices AB, BA, AC, CA if they are defined. If they are not defined, explain why.

2. Set A to be the matrix

$$A = \begin{pmatrix} 1 & -1 & 1 \\ -1 & 3 & 1 \\ 2 & 2 & 6 \end{pmatrix}$$

   (a) Factor $A = LU$ where L is lower triangular and U is upper triangular.

   (b) Find the row reduced echelon form R of A.

   (c) Find a non-zero vector in the null space of A or explain why none exists.

3. Find a system of equations whose solutions consist of the line through the origin and the vector $(1, , 1, 1)^T$ in $\mathbb{R}^3$.

4. Set

$$A = \begin{pmatrix} 1 & -1 & 1 \\ -1 & 3 & 1 \\ 2 & 2 & 6 \end{pmatrix}, \qquad b = \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix}$$

   (a) Find a solution of the equation $Ax = b$.

   (b) Is the set of solutions of $Ax = b$ a subspace? Why or why not?