Name:_____

communicate: show work and indicate reasons

1a) Solve the system $A\vec{x} = \vec{b}$ by elimination on the augmented matrix $[A \ \vec{b}]$, where

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 3 & -1 \\ 0 & 4 & 2 \end{bmatrix}, \qquad \vec{b} = \begin{bmatrix} 0 \\ 1 \\ 7 \end{bmatrix}.$$

1b) Use the calculations you have done above, to write lower and upper triangular matrices for which $A = LU$.

2) $B = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$. Find B^{-1} and use it to solve

$$B\vec{x} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

3a) Explain why the nullspace of a matrix A is closed under scalar multiplication, that is, if $\vec{v}$ is in the nullspace of A and c is a number, why is $c\vec{v}$ also in the nullspace?

3b) A matrix B has been reduced by row operations to

$$R = \begin{bmatrix} 1 & -1 & 0 & 5 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Find all solutions to $B\vec{x} = \vec{0}$.

4) $\vec{w}_1 = \begin{bmatrix} k \\ 1 \\ 1 \end{bmatrix}$, $\vec{w}_2 = \begin{bmatrix} 1 \\ k \\ 1 \end{bmatrix}$, $\vec{w}_3 = \begin{bmatrix} 1 \\ 1 \\ k \end{bmatrix}$.

Find k so that any two of these vectors are orthogonal.

Then find the length $||\vec{w}_1||$ and the distance $||\vec{w}_2 - \vec{w}_3||$.

some short answers:

1. $\vec{x} = \begin{bmatrix} \frac{3}{4} \\ \frac{3}{4} \\ 2 \end{bmatrix}$

2. $\vec{x} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

3. there are two pivots and two free variables

4. $||\vec{w}_1|| = 3/2$