Math 2310 Prelim 1, Feb. 26, 2014 **Name:**

Work + Communication = Credit

1. Here are three linear equations in three variables:

$$\begin{aligned} x + 2y - z &= 2 \\ y + 3z &= 0 \\ 2x + 5y + z &= 4. \end{aligned}$$

Describe the set of solutions as a line, point, plane, empty, etc. Use any method you want. Show all work. Just an answer, even a correct one is worth ZERO points.

2. Let A be the 3×3 matrix

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & 3 \\ 1 & 1 & 8 \end{bmatrix}.$$

Find the LU decomposition of A. Use your decomposition to solve $A \cdot \vec{x} = \begin{bmatrix} -3 \\ 2 \\ 9 \end{bmatrix}$. Solving directly from A is worth ZERO points.

3. For this problem $\vec{u}$ and $\vec{v}$ are perpendicular unit vectors in $\mathbb{R}^4$. In addition, $\vec{w} = a\ \vec{u} + b\ \vec{v}$ is a linear combination of $\vec{u}$ and $\vec{v}$.

   (a) Compute $\vec{w} \cdot \vec{u}$ and $\vec{w} \cdot \vec{v}$ in terms of a and b.

   (b) Calculate $||2\vec{u} + 3\vec{v}||$.

4. A is a 3×3 matrix and a solution to $A \cdot \vec{x} = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$ is $\begin{bmatrix} -4 \\ 0 \\ -6 \end{bmatrix}$.

   For what values of k could A^{-1} equal

$$\begin{bmatrix} 2 & k & k \\ \sqrt{317} & 2 & 1 \\ 3 & 6 & 6 \end{bmatrix}?$$

5. Let A be a 3×5 matrix. Suppose $\vec{z}$ is in the null space of A and $\vec{v_1}$ is a solution of the matrix equation $A \cdot \vec{x} = \vec{b}$. Is $\vec{v_1} + \vec{z}$ always a solution of the matrix equation $A \cdot \vec{x} = \vec{b}$? If not, give a counterexample. If so, explain why.