MATH 2310 — Prelim 2

November 5, 2012

Name: _____ Lecture: _____

- Do not open this booklet until instructed to begin.

- You will have a total of 50 minutes to complete the exam, which consists of 5 problems. Please show work and/or justification if asked (although doing so even when not asked may accumulate some partial credit). Each problem is weighted equally. Books, notes, calculators, cell phones, and other forms of assistance are not to be used during the exam.

- Each problem appears on its own page. Please feel free to use the back of the page to continue your work; if you require additional paper, raise your hand and I will supply some. Label clearly which problem appears on the additional page(s) and indicate the final answer.

Problem	Grade	Possible
1		20
2		20
3		20
4		20
5		20
Total		100

Problem 1:
Consider the two (not quite identical) matrices

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 2 \\ 1 & 0 & 0 & 1 \end{bmatrix}.$$

(a) Compute $\det(A)$.

(b) Compute $\det(B)$.

Problem 2:
Consider the matrix

$$D = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 2 & 2 \\ 0 & 0 & 0 \\ 2 & 1 & 1 \end{bmatrix}$$

(a) Find a basis for the column space $C(D)$.

(b) Find an orthonormal basis for the row space $R(D)$.

(c) Find a basis for the orthogonal complement $R(D)^{\perp}$ of the row space.

Problem 3:

True/False section! No written justification is required; just circle T or F (but not both!). Each correct answer is worth two points, each blank answer is worth one point, and each incorrect answer is worth no points.

T F The dimensions of the row space $R(A)$ and the column space $C(A)$ are equal for *any* (not necessarily square) matrix A.

T F A square matrix is singular if and only if it has 0 as an eigenvalue.

T F The vector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is an eigenvector of the matrix $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$.

T F The set of vectors orthogonal to a given nonzero vector $\mathbf{v} \in \mathbb{R}^n$ forms a subspace of dimension $n - 1$.

T F Any subspace spanned by three vectors has dimension 3.

T F The only vector in $\mathbb{R}^3$ orthogonal to all three of $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is the zero vector.

T F Putting a square matrix into reduced row echelon form does not change its determinant.

T F At least four data points are required to fit a straight line by least squares approximation.

T F If A is a square matrix, then $\det(2A) = 2\det(A)$.

T F If B is an invertible square matrix, then $\det(B^{-1}) = \frac{1}{\det(B)}$.

Problem 4:
Consider the matrix
$$M = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$$

(a) Find, if possible, two linearly independent eigenvectors of M.

(b) For each eigenvector $\mathbf{v}$ found in part (a), sketch a graph of both $\mathbf{v}$ and $M\mathbf{v}$ in $\mathbb{R}^2$.

Problem 5:

Find an equation for the line which best fits (in the sense of least squares approximation) the data set $\{(0, 3), (1, 1), (2, 2), (3, 1)\}$. These data points are given in the format (x, y) (or, if you prefer the book's notation, (t, b)).