Name:________________________________

Test 2 Math 2310 Spring 2013

communicate: show work and indicate reasons

1a) Find all solutions to the system $A\vec{x} = \vec{b}$, where

$$A = \begin{bmatrix} 1 & 0 & -1 & 2 & 1 \\ 2 & 1 & 1 & 4 & 0 \end{bmatrix}, \qquad \vec{b} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}.$$

1b) Use calculations you have done above to give a basis for the null space of A and state the dimension of the null space.

2a) What properties should a list of vectors have, so that they are a basis for a vector space?

2b) The set of vectors of the form $\begin{bmatrix} a+b \\ a-b \\ b \end{bmatrix}$, where a and b can be any numbers, is a vector space. Find a basis, explaining why your list of vectors has the required properties.

Bonus question 2 points only: State the values of t for which the following expression is meaningful, and simplify it as much as possible:

$$\frac{t}{1-t} + \left(1 + \frac{1}{t} + \frac{1}{t^2} + \frac{1}{t^3} + \frac{1}{t^4} + \cdots\right)$$

3) Here $A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 1 \\ 0 \\ -2 \\ 0 \end{bmatrix}$.

a) Explain by inspection of A and $\vec{b}$ why $A\vec{x} = \vec{b}$ has no solution.

b) Find the point of the column space $C(A)$ closest to $\vec{b}$.

4) Here $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ and $\vec{a} = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$.

a) Show that the columns of A are linearly independent.

b) Verify that the orthogonal complement of $C(A)$ consists of all scalar multiples of $\vec{a}$.

c) What is the relation between the projections $\vec{a}(\vec{a}^T\vec{a})^{-1}\vec{a}^T$ and $A(A^TA)^{-1}A^T$? [You are not asked to calculate them.]

some short answers:

1a. one solution is $\begin{bmatrix} 5 \\ -3 \\ 0 \\ 0 \\ 0 \end{bmatrix}$.

1b. the dimension is 3, because there are 3 basis vectors

2. the dimension is 2, because there are 2 basis vectors. The vectors $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ are independent because if the combination

$$a \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + b \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} a+b \\ a-b \\ b \end{bmatrix}$$

were $\vec{0}$, then the third component would give $b = 0$, and then the first or second would give $a = 0$. That's the definition of independent.

3b. $\begin{bmatrix} -\frac{1}{2} \\ 0 \\ -\frac{1}{2} \\ 0 \end{bmatrix}$

4a. because there are 3 pivots when you reduce

4c. their sum is the 4 by 4 identity matrix I