Math 2310 Prelim 2, April 9, 2014 **Name:**

Work + Communication = Credit

1. (a) True/False: The rank of a matrix equals the dimension of its row space. (No explanation needed.)

   (b) If A is a 3×6 matrix and $A \cdot \vec{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ has no solution, then the dimension of the null space of A is at least ____. Give a short explanation of your answer.

   (c) Suppose P is an $n \times n$ projection matrix. Then $||P \cdot \vec{x}|| \leq ||\vec{x}||$ for all $\vec{x}$ in $\mathbb{R}^n$. Why?

   (d) Pat overhears another math 2310 student say that A is a 4×4 matrix and $A \cdot \begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \\ -2 \end{bmatrix}$ and $A \cdot \begin{bmatrix} 2 \\ -2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 1 \\ 0 \end{bmatrix}$. Immediately Pat says, "Oh, A is not an orthogonal matrix". How did Pat know this?

2. Write down the right-hand-side of the matrix equation used to find the best fit of the data

x	y
-2	1
-1	2
1	3
2	1
3	4

to an equation of the form

$$y = \frac{C}{x^2} + \frac{D}{x} + E \cdot x.$$

$$\begin{bmatrix} C \\ D \\ E \end{bmatrix} =$$

Do NOT compute an answer!!

3.

4. For this problem A is the 3×5 matrix

$$\begin{bmatrix} -2 & 8 & 1 & -4 & 5 \\ 1 & -4 & 0 & 2 & -1 \\ 2 & -8 & 1 & 4 & 1 \end{bmatrix}.$$

   (a) Find a basis for the row space of A.

   (b) Find a basis for the column space of A.

   (c) Find a basis for the null space of A.

   (d) What is the dimension of the left null space of A?

   (e) Write down the complete solution to $A\,\vec{x} = \begin{bmatrix} 0 \\ 1 \\ 4 \end{bmatrix}$.

   (Use the back of this page if necessary)