

MATH 2310 QUIZ

Friday 2 October 2009. You have 50 minutes. No calculators are permitted. Please show all working.

- (1) True or False? (Explain your answer.)
- (a) No linear system has exactly two solutions.
 - (b) If A is *any* $n \times n$ matrix, then the matrix $I_n - AA^T$ is symmetric. (Here, I_n denotes the identity matrix of size $n \times n$.)
- (2) Let

$$A = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 4 & 6 \\ 0 & 1 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 1 \\ -1 & 0 \\ 0 & 2 \end{bmatrix}$$

Calculate each of the following. If it is not defined, say so.

- (a) $A(C + B^T)$
 - (b) B^{-1}
- (3) Let $X = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$.
- (a) Show that $X^2 - 7X + I_2 = 0$.
 - (b) Determine whether X is invertible, and find X^{-1} if it exists.
- (4) Let

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 9 \\ 3 & 1 & 12 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

- (a) Find all solutions to the linear system $A\mathbf{x} = \mathbf{b}$.
 - (b) Is A invertible? Explain why or why not.
- (5) Determine whether the matrix

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -2 \\ 0 & -1 & 2 \end{bmatrix}$$

is invertible, and find A^{-1} if it exists.

[END]