HW2 Solutions

Math 2310

1.3.1

$$3\begin{bmatrix}1\\1\\1\end{bmatrix}+4\begin{bmatrix}0\\1\\1\end{bmatrix}+5\begin{bmatrix}0\\0\\1\end{bmatrix}=\begin{bmatrix}3+0+0\\3+4+0\\3+4+5\end{bmatrix}=\begin{bmatrix}3\\7\\12\end{bmatrix}=b.$$

$$b=Sx=\begin{bmatrix}1&0&0\\1&1&0\\1&1&1\end{bmatrix}\begin{bmatrix}3\\4\\5\end{bmatrix}=\begin{bmatrix}(1,0,0)\cdot x\\(1,1,0)\cdot x\\(1,1,1)\cdot x\end{bmatrix}=\begin{bmatrix}3\\7\\12\end{bmatrix}.$$

1.3.8

$x_1=b_1$. $-x_1+x_2=b_2$, so $x_2=b_1+b_2$. $-x_2+x_3=b_3$, so $x_3=b_1+b_2+b_3$. $-x_3+x_4=b_4$, so $x_4=b_1+b_2+b_3+b_4$. These assemble into

$$x=\begin{bmatrix}b_1\\b_1+b_2\\b_1+b_2+b_3\\b_1+b_2+b_3+b_4\end{bmatrix}=\begin{bmatrix}1&0&0&0\\1&1&0&0\\1&1&1&0\\1&1&1&1\end{bmatrix}\begin{bmatrix}b_1\\b_2\\b_3\\b_4\end{bmatrix}$$

1.3.14

If (a,b) is a multiple of (c,d), then for some e we have $a=ec$ and $b=ed$. This means $(a,c)=(ec,c)=c(e,1)$ and $(b,d)=(ed,d)=d(e,1)$, so we have $\frac{d}{c}(a,c)=\frac{cd}{c}(e,1)=d(e,1)=(b,d)$.

2.1.5

If x,y,z satisfy the first two equations then they also satisfy the third. Some solutions on the line: $(0,1,1)$, $(1,1,0)$, $(2,1,-1)$.

2.1.8

Four 3-dimensional planes in 4-dimensional space typically meet at a point, as each plane is defined by a linear equation so their intersection is described by 4 equations in 4 variables, which typically have a single point as a solution.

2.1.9b

$$\begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} (2,1,0,0)\cdot(1,1,1,2) \\ (1,2,1,0)\cdot(1,1,1,2) \\ (0,1,2,1)\cdot(1,1,1,2) \\ (0,0,1,2)\cdot(1,1,1,2) \end{bmatrix} = \begin{bmatrix} 2+1+0+0 \\ 1+2+1+0 \\ 0+1+2+2 \\ 0+0+1+4 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 5 \end{bmatrix}$$

2.1.10b

$$\begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 2 \\ 1 \end{bmatrix} + 2 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2+1+0+2*0 \\ 1+2+1+2*0 \\ 0+1+2+2*1 \\ 0+0+1+2*2 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 5 \end{bmatrix}$$

2.1.12

$$\begin{bmatrix} z \\ y \\ x \end{bmatrix}$$

$$\begin{bmatrix} 2+1-3 \\ 1+2-3 \\ 3+3-6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2+1 \\ 1+2 \\ 3+3 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 6 \end{bmatrix}$$

2.1.31

$$M_3 = \begin{bmatrix} 8 & 3 & 4 \\ 1 & 5 & 9 \\ 6 & 7 & 2 \end{bmatrix}.$$

$M_3(1,1,1) = (15,15,15)$ for any choice of M_3. M_4 should be defined so each row sums to $(1 + ... + 16)/4 = 136/4 = 34$, so we would have M_4 times (1,1,1,1) as (34,34,34,34).

2.2.8

$k = 0$ is fixed by a row exchange and has 1 solution.

$k = 3$ is not fixed by a row exchange and has no solutions as $3x + 3y$ cannot be both 6 and -6.

$k = -3$ is not fixed by a row exchange and has ∞ solutions as the second equation is just the first one scaled on both sides by -1, and $-3x + 3y = 6$ has infinitely many solutions.

2.2.11

(a) If A is a 3x3 matrix and both $A(x,y,z) = b$, $A(X,Y,Z) = b$, then

$$A(\frac{1}{2}(x,y,z) + \frac{1}{2}(X,Y,Z)) = \frac{1}{2}A(x,y,z) + \frac{1}{2}A(X,Y,Z) = \frac{1}{2}b + \frac{1}{2}b = b$$

so $\frac{1}{2}(x,y,z) + \frac{1}{2}(X,Y,Z)$ is another solution.

(b) The planes also meet everywhere on the unique line containing both of the two points.

2.2.12

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

2.2.13

Subtract 2 × row 1 from row 2

Subtract 1 × row 1 from row 3

Subtract 2 × row 2 from row 3

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$$