HW8 Solutions

Math 2310

5.2.1 (first matrix)

$det(A) = 1 + 12 + 18 - 4 - 6 - 9 = 12$, so the rows of A are independent.

5.2.4 (first matrix)

$a_{11}a_{23}a_{32}a_{44}$ gives -1, because the terms $a_{23}a_{32}$ have columns 2 and 3 in reverse order. $a_{14}a_{23}a_{32}a_{41}$ which has 2 row exchanges gives +1, so $det(A) = 1 - 1 = 0$.

5.3.1a

$det(A) = 2 * 4 - 5 * 1 = 3$.

$x_1 = \frac{1}{3} det \begin{bmatrix} 1 & 5 \\ 2 & 4 \end{bmatrix} = \frac{4-10}{3} = -2.$

$x_2 = \frac{1}{3} det \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = \frac{4-1}{3} = 1.$

5.3.6a

$det(A) = 3$. $C^T = \begin{bmatrix} 3 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & -7 & 3 \end{bmatrix}.$

$A^{-1} = \frac{1}{3} \begin{bmatrix} 3 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & -7 & 3 \end{bmatrix}.$

5.3.15

For $n = 5$ the matrix C contains *25* cofactors. Each 4 by 4 cofactor contains $4! = 24$ terms and each term needs 3 multiplications (since there are 4 things being multiplied). Therefore we do 24*3*25=1875 multiplications (and some additions), so this calculation of the inverse is much slower than using Gauss-Jordan.

5.3.16

(a) $det \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = 10.$

(b-c) Each of these triangles is half of the parallelogram in (a), so they both have area 5.

6.1.2

$det(A - \lambda I) = (1 - \lambda)(3 - \lambda) - 8 = \lambda^2 - 4\lambda - 5 = (\lambda + 1)(\lambda - 5) = 0$, so $\lambda_1 = -1$, $\lambda_2 = 5$. The eigenvectors are $x_1 = (-2, 1)$, $x_2 = (1, 1)$. $A + I$ has the *same* eigenvectors as A. Its eigenvalues are *increased* by 1 (so 0 and 6).

6.1.3

A has $\lambda_1 = 2$ and $\lambda_2 = -1$ with $x_1 = (1, 1)$ and $x_2 = (2, -1)$. A^{-1} has the *same* eigenvectors, with eigenvalues $1/\lambda = \frac{1}{2}$ and -1.

6.1.5

A and B have eigenvalues 1 and 3 (their diagonal entries as they are triangular matrices). $A + B$ has $\lambda^2 + 8\lambda + 15 = 0$ and $\lambda_1 = 3$, $\lambda_2 = 5$. Eigenvalues of $A + B$ *are not equal* to eigenvalues of A plus eigenvalues of B.

6.1.6

A and B have $\lambda_1 = 1$ and $\lambda_2 = 1$. AB and BA have $\lambda^2 - 4\lambda + 1 = 0$ and the quadratic formula gives $\lambda = 2 \pm \sqrt{3}$. Eigenvalues of AB *are not equal* to eigenvalues of A times eigenvalues of B. Eigenvalues of AB and BA *are equal*.

6.1.17

$\lambda_1 = \frac{1}{2}(a + d + \sqrt{(a-d)^2 + 4bc})$ and $\lambda_2 = \frac{1}{2}(a + d - \sqrt{\ })$ add to $a + d$. If A has $\lambda_1 = 3$ and $\lambda_2 = 4$ then $det(A - \lambda I) = (\lambda - 3)(\lambda - 4) = \lambda^2 - 7\lambda + 12$.

6.1.21

$(A - \lambda I)$ and $(A^T - \lambda I)$ have the same determinant as transpose preserves determinant, $I^T = I$ and transpose preserves addition and scaling of matrices. However, the following matrices have different eigenvectors:

$\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ has eigenvectors $(1, 1)$ and $(0, 1)$ while

$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ has eigenvectors $(1, 0)$ and $(1, -1)$.

6.1.24

The eigenvectors are 0 (multiplicity 2) and 6. The eigenvectors are $(0, -2, 1)$, $(1, -2, 0)$ (both perpendicular to $(1, 2, 1)$), and $(1, 2, 1)$.

6.1.29

A) 1,4,6

B) 2, $\sqrt{3}$, $-\sqrt{3}$

C) 0,0,6

6.1.30

$A(1,1) = (a+b, c+d) = (a+b, a+b) = (a+b)*(1,1)$, so $a+b$ is an eigenvalue. To preserve the trace, the other eigenvalue must be $d-b$.

6.2.1

(a) $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 3/4 & -1/4 \\ 1/4 & 1/4 \end{bmatrix}$

(b) $A^3 = X\Lambda^3 X^{-1}$ and $A^{-1} = X\Lambda^{-1}X^{-1}$

6.2.2

$A = X\Lambda X^{-1} = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}$.

6.2.11

(a) True (no zero eigenvalues)

(b) False (repeated $\lambda = 2$ may have only one line of eigenvectors)

(c) False (repeated λ may have a full set of eigenvectors)

6.2.15

$A^k = X\Lambda^k X^{-1}$ approaches 0 if and only if every λ has absolute value less than 1. This is the case for A_2 but not A_1.

6.2.18

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \text{and}$$

$$A^k = \frac{1}{2} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 3^k \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1+3^k & 1-3^k \\ 1-3^k & 1+3^k \end{bmatrix}$$

6.2.25

For each column a_i of A, $Aa_i = a_i$, so the columnspace contains eigenvectors for $\lambda = 1$. Eigenvectors for $\lambda = 0$ satisfy $Ax = 0x = 0$, and vectors x with this property make up the nullspace.