MATH 2310 Linear Algebra with Applications
Fall 2019 Exam #1

INSTRUCTIONS

- You have 75 minutes. If you finish within the last 15 minutes of class, please remain seated until the end so as not to disturb your classmates.
- The exam is closed book, closed notes, no calculators/computers/etc.
- Mark your answers ON THE EXAM ITSELF. If you are not sure of your answer, you may wish to provide a *brief* explanation so that we can at least know what you are trying to do. All short answer questions can be successfully answered in a few sentences at most. For full credit, be sure to show your work and justify your steps. Little credit will be given for correct answers without justification.
- Questions are not given in order of difficulty. Make sure to look ahead if stuck on a particular question.

Last Name	
First Name	
Cornell NetID (e.g. bwh59)	
All the work on this exam is my own. **(please sign)**	

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Q. 1	Q. 2	Q. 3	Q. 4	Q.5	Total
/20	/20	/20	/ 20	/20	/100

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1. (20 points) True/False

If true, give a short justification for why. If false, produce a counterexample and show why it contradicts the statement.

(a) If A is invertible, so is its transpose A^T.

(b) If $A = A^T$, then A is invertible.

(c) Given any vector $\vec{w} \in \mathbf{R}^3$, the set of all vectors $\vec{v} \in \mathbf{R}^3$ perpendicular to $\vec{w}$ (i.e. $\vec{w} \cdot \vec{v} = 0$) forms a subspace.

(d) Given any system of equations $A\vec{x} = \vec{b}$, the set of solutions $\{\vec{x} \mid A\vec{x} = b\}$ to the system, *if nonempty*, is a subspace.

(e) Let A be any matrix. If its nullspace $N(A) = \{\vec{x} \mid A\vec{x} = \vec{0}\}$ is equal to $\{\vec{0}\}$, then there must exist exactly one solution to $A\vec{x} = \vec{b}$.

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2. (20 points) Solving Systems of Linear Equations

(*Tip*: These are long-ish calculations; be smart about how you proceed and be neat and organized with your work. You almost certainly want to start on scratch paper.)

(a) (10 points) Find all solutions $\vec{x} = (x_1, x_2, x_3, x_4)$ to the following system of equations:

$$\begin{aligned} x_1 + 2x_2 + x_3 + x_4 &= 0 \\ 3x_1 + 4x_2 + 2x_3 + x_4 &= 0 \\ 4x_1 + 6x_2 + 3x_3 + 2x_4 &= 0 \end{aligned}$$

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(b) (10 points) Find all solutions $\vec{x}$ to the following system of equations:

$$\begin{aligned} x_1 + 2x_2 + x_3 + x_4 &= 1 \\ 3x_1 + 4x_2 + 2x_3 + x_4 &= 2 \\ 4x_1 + 6x_2 + 3x_3 + 2x_4 &= 3 \end{aligned}$$

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3. (20 points) LU Factorization

We want to find the "LU factorization" of the following 2×4 matrix:

$$A = \begin{bmatrix} 1 & 1 & 2 & 2 \\ 2 & 2 & 3 & 1 \end{bmatrix}$$

(a) (5 points) How many rows and columns should the "lower triangular" matrix L have?

(b) (5 points) How many rows and columns should the "upper triangular" matrix U have?

(c) (10 points) Find the LU factorization of A.

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4. (20 points) The Number of Solutions to $A\vec{x} = \vec{b}$.

Given a matrix A of the following type, what are the possible numbers of solutions $\vec{x}$ to equations of the form $A\vec{x} = \vec{b}$? Be sure to justify your answers (e.g. with specific examples).

For example, if A is a 1×1 matrix, so $ax = b$ for some scalars $a, b \in \mathbf{R}$, we can have 0 solutions (e.g. $0 \cdot x = 1$), 1 solution (e.g. $2x = 3$), or infinitely many solutions (e.g. $0 \cdot x = 0$).

(a) (5 points) A a 3×3 matrix.

(b) (5 points) A a 2×4 matrix.

(c) (5 points) A a 4×3 matrix.

(d) (5 points) A a 4×4 matrix with at least one row of all zeros.

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5. (20 points) **Finding Inverses.** Consider the matrix

$$A = \begin{bmatrix} 3 & 4 & 1 \\ 2 & 3 & 6 \\ 1 & 2 & 1 \end{bmatrix}.$$

(a) (10 points) Without explicitly calculating the inverse, show that A is invertible by using one of the criteria we have for invertibility.

(b) (10 points) Find the inverse A^{-1} of A. (This is a long computation, but relatively straightforward if you do it right, so be smart and organized with your work. You almost certainly want to try it on scratch paper first. Be sure to *check your answer* if you have time.)

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