MATH 2310 Linear Algebra with Applications
Fall 2019 Exam #2

INSTRUCTIONS

- You have 75 minutes. If you finish within the last 15 minutes of class, please remain seated until the end so as not to disturb your classmates.
- The exam is closed book, closed notes, no calculators/computers/etc.
- Mark your answers ON THE EXAM ITSELF. If you are not sure of your answer, you may wish to provide a *brief* explanation so that we can at least know what you are trying to do. All short answer questions can be successfully answered in a few sentences at most. For full credit, be sure to show your work and justify your steps. Little credit will be given for correct answers without justification.
- Questions are not given in order of difficulty. Make sure to look ahead if stuck on a particular question.

Last Name	
First Name	
Cornell NetID (e.g. bwh59)	
All the work on this exam is my own. **(please sign)**	

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Q. 1	Q. 2	Q. 3	Q. 4	Q.5	Total
/20	/20	/20	/ 20	/20	/100

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1. (20 points) Five True/False Questions on Eigenvalues

If true, give a short justification for why. If false, produce a counterexample and show why it contradicts the statement.

(a) If a square matrix A has 0 as an eigenvalue, then it is not invertible.

(b) If $AB = BC$ for some matrix B, then A and C have the same eigenvalues.

(c) If two $n \times n$ matrices A and C have the same eigenvalues, then they are similar.

(d) Let A be an $n \times n$ matrix whose entries are integers. All the eigenvalues of A must be real numbers.

(e) Let A be an $n \times n$ matrix that is diagonalizable whose eigenvalues are $\lambda_1, \ldots, \lambda_n$ (not necessarily distinct). Then A^{100} has eigenvalues $\lambda_1^{100}, \lambda_2^{100}, \ldots, \lambda_n^{100}$.

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2. (20 points) Determinant

Calculate the determinant of

$$A = \begin{bmatrix} 1 & 0 & 3 & 4 \\ 0 & 2 & 0 & 3 \\ 1 & 1 & 2 & 0 \\ 3 & 1 & 2 & 5 \end{bmatrix}.$$

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3. (20 points) Orthonormal Basis and QR Factorization

Consider the vectors

$$\vec{v_1} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad \vec{v_2} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \quad \vec{v_3} = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 7 \end{bmatrix}$$

(a) Find an orthonormal basis for $\text{span}\{\vec{v_1}, \vec{v_2}, \vec{v_3}\}$ via the Gram–Schmidt process.

(b) Produce a decomposition

$$\begin{bmatrix} \vec{v_1} \ \vec{v_2} \ \vec{v_3} \end{bmatrix} = QR$$

where Q is an orthogonal matrix (i.e. $Q^{-1} = Q^T$) and R is an upper-triangular matrix.

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4. (20 points) Powers of a Matrix

Consider the matrix

$$A = \begin{bmatrix} -1 & -1 & 1 \\ 6 & 4 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

(a) (10 points) What are the eigenvalues and eigenvectors of A?

(b) (10 points) What is A^{100}?

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5. (20 points) Orthogonal Projections.

Consider the plane in $\mathbf{R}^3$ given by

$$U = \text{span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

(a) What is the matrix P that gives the orthogonal projection of a vector $\vec{v} \in \mathbf{R}^3$ onto U? Check that $P^2 = P$ and $P = P^T$.

(b) What is the closest point on U to $\vec{v} = \begin{bmatrix} 11 \\ 7 \\ 19 \end{bmatrix}$? Justify why your answer is the closest point.

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