CLASS QUESTIONS ON GRAPHS

Like we've been practicing in class, the "full answer" to one of these questions is not just answering the literal question itself, but to, say, play around with the hypotheses and results and look at patterns in the proofs to try and find a *general answer* to the question that's being asked. In a sense, you're trying to answer the question that's *really* being asked when the question is posed, as opposed to just interpreting things literally, like a robot.

Of course, this means that there are possibly many potential ways to fully answer the questions, but as long as you have taken it a few steps further beyond the question itself, that is fine.

(1) Can you draw a connected planar graph with 5 vertices and more than 9 edges?
    Example question to answer: Can you guess the general rule, if there are n vertices, how many edges can you impose so that your graph is still planar?
(2) What are the conditions (if there are any) on the number of vertices v, the number of edges e, and number of faces f for graph G to be a tree?
(3) Which dimension of a hyper cube graph is planar?
(4) Is it possible to come up with some way to count faces given a non-planar graph? Do you have to institute some kind of regulation or requirement for the way the edges would have to be arranged to get certain standard amounts of faces from a non-planar graph?
(5) How many paths are there from one vertex to another in a connected path without using the same edge twice?
(6) How many distinct graphs can be created from a set of v vertices? How many connected graphs? How many forests? How many trees?
(7) For a connected planar graph, how many paths are possible?
(8) How can we choose a set of vertices with the maximum possible number of elements such that every vertex in the set is not adjacent to any of the other vertices in the set?
(9) A binary tree is a tree that for every parent node, there are 2 children nodes. For a binary tree of height h, show that the number of nodes n is always less than or equal to $2^{h+1} - 1$.
(10) Is there a bound on the chromatic number for nonplanar tree graphs?
(11) How might you prove Euler's formula by inducting on edges or vertices as opposed to faces?
(12) Given a connected graph that is non planar such that $|V| \leq n$, what would be maximal number of ways in which the graph can cross, i.e. its edges can cross?