MATH 3040: HOMEWORK 10

All answers must be in complete sentences. All assertions must be proven, per the student contract. Remember that this is a writing class, so while correctness and thoroughness are most important, *style* matters as well.

1. (Practice in Posing Questions) What is an interesting open-ended question you could ask, which is related to the material that we covered in class? What is a concrete mathematical question that could be used to approach an answer to the question?

2. (Wrap-Up for Planes sheet)

(a) (Question 9.2) Describe the smallest projective plane. Why must it be the smallest one?

(b) (Duality) Show that if you have a projective plane $\Pi = (\mathcal{P}, \mathcal{L})$, then $\Pi^D = (\mathcal{L}, \mathcal{P})$, where points of Π^D are the lines of Π and vice-versa, is also a projective plane.

Cardinalities seemed to be the trickiest topic on Prelim II, so in addition to our next sheet which deals with the topic, here are a couple of addition questions to solidify our knowledge. Refer back to our sheet on Infinities.

3. (Concrete Uncountability) These are primarily testing your understanding of the argument of *Infinities, Theorem 10.*

(a) (Infinities, Question 11) Let S be the set of infinite sequences of 0's and 1's. Show that S is uncountable.

(b) Show that if a and b are real numbers with $(a < b)$, then the open interval (a, b), the closed interval $[a, b]$, and the half-closed intervals $(a, b]$ and $[a, b)$ are all uncountable. (*Hint:* Reduce to the "smallest" case)

4. (Abstract Infinity and Uncountability)

(a) Show that if $A \subset B$ and A is uncountable, then B is uncountable.

(b) If A is uncountable and B is countable, then $A \backslash B$ is uncountable.

(c) Let A be uncountable and B be an arbitrary set. Which of the following statements is true? (If true, prove it; if false, provide a counterexample.)

(i) The set $A \cup B$ is uncountable.

(ii) The set $A \cap B$ is uncountable.

(iii) The set $A \times B$ is uncountable.

(iv) The set $A \backslash B$ is uncountable.