MATH 3040: HOMEWORK 2

All answers must be in complete sentences. Remember that this is a writing class, so while correctness and thoroughness are most important, *style* matters as well. If you'd like, you can use the `hw_template.tex` file to set up your LaTeX document.

Do not start a sentence with a symbol, and there should be words in-between your math symbols. For example, "If $x = 2$, $2 \cdot x = 4$" is not good, you should write "If $x = 2$, then $2 \cdot x = 4$" instead.

1. (Practice in Posing Questions) What is an interesting open-ended question you could ask, which is related to the material that we covered in class? What is a concrete mathematical question that could be used to approach an answer to the question? (If you also want to provide a proof of this concrete mathematical question, that's great and we'll look at it, but what you're being graded on are the questions themselves.)

Your answer should be a sort of mini-essay.

Here's an example of the kind of thing I'm looking for:

> In class, we divided numbers into even and odd numbers, which are a familiar notion, but which correspond to divisibility by 2. What if, given an integer k, we divided the set $\mathbf{Z}$ of all integers into k groups, with respect to divisibility by k? For example, if $k = 3$, suppose that we divide $\mathbf{Z}$ into three groups according to their remainder, so that we have one group consisting of multiples or 3, one group consisting of integers m such that $m - 1$ is divisible by 3, and one group consisting of integers m such that $m - 2$ is divisible by 3. Which of the statements on the Numbers sheet about the parity of numbers (i.e. Exercises 3–5) generalize to this context? What about if we divided $\mathbf{Z}$ up into 3 parts in some other way?
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> From one perspective, the essence of the statement seems to be this: *What properties of "2" are really being used in the proofs?* Is it the fact that it divides $\mathbf{Z}$ into 2? Or maybe the fact that it is an integer, that it is finite, or that it is a prime number?
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> A concrete mathematical question that could be used to work towards an answer is to try some of the Exercises 3–5 in the case of $k = 3$, before trying other k. For example, generalizing Exercise 4.3, is it the case that if $a, b \in \mathbf{Z}$, the

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number $a + b$ is divisible by 3 if and only if $a - b$ is divisible by 3?

2. (Prove or Disprove and Salvage) For the three following statements, either (a) prove, or (b) disprove and salvage. Proving is self-explanatory. You disprove by providing a counterexample and showing how it is a counterexample, and you salvage by correcting the statement and proving the corrected statement. If you are salvaging a statement, be as general as possible if you want full credit. For example, the false statement "All numbers are divisible by two" should be corrected to "All *even* numbers are divisible by two," as opposed to "The number 4 is divisible by two" or "Multiples of 8 are divisible by two."

(a) The sum of any three consecutive integers is always divisible by 3.
(b) Let $a, b, m \in \mathbf{Z}$. If ab divides m, the a divides m and b divides m.
(c) Let $x \in \mathbf{Z}$. If x^2 is even, then x is even.

3. (Practice in Reading, applying Axioms, etc.) Read Chapter 1 of Beck-Geoghegan and prove Proposition 1.27 (ii) and (iii), following the book's model for Prop. 1.27(i).

For either (ii) or (iii) (your choice), write the same proof in two ways: one in the style of the book (few words, writing exactly what Propositions and axioms are being used, in the spirit of the two-column proofs from high school geometry class) and one in the style that I would like you to write in the course (complete sentences, saying less, but doing it in a manner that would convince both you and the reader that you know what you're doing).

4. (Groupwork: Bridge Standoff) (To be submitted as *one assignment* by the groups we formed in the first week. Use `groupwork_template.tex`.) Submit your answers with complete proofs.

(a) Which player has the winning strategy if the bridge is of length n, where n is an arbitrary positive integer?
(b) Now suppose that instead of only being allowed to jump 1, 2, or 3, you are allowed to jump any number of feet from 1 to an arbitrary integer k. (But still, you *must* jump each turn!) For all possible pairs of positive integers (n, k) (="(length, step size)"), who has the winning strategy?