MATH 3040: HOMEWORK 8

All answers must be in complete sentences. All assertions must be proven, per the student contract. Remember that this is a writing class, so while correctness and thoroughness are most important, *style* matters as well.

1. (Practice in Posing Questions) What is an interesting open-ended question you could ask, which is related to the material that we covered in class? What is a concrete mathematical question that could be used to approach an answer to the question?

2. (Wrap-up from Relations) Let A be the set of 5 elements (not 4 like the sheet). How many relations are there on A? Symmetric relations? Functions from A to A? Equivalence relations?

3. (Prove or Disprove and Salvage) Let S be a nonempty set, and suppose that R_1 and R_2 are relations on S.

(a) If R_1 and R_2 are symmetric, then $R_1 \cup R_2$ is symmetric.
(b) If R_1 or R_2 is transitive, then $R_1 \cup R_2$ is transitive.
(c) If R_1 or R_2 is reflexive, then $R_1 \cap R_2$ is reflexive.

4. (Equivalence Relations)

(a) Let A be a non-empty set and B a fixed subset of A. We define a relation on the powerset $\mathcal{P}(A)$ of A by $X \sim Y$ if $X \cap B = Y \cap B$. Show that this is an equivalence relation.
(b) Define a relation on the integers as follows: $a \sim b$ if $a + b$ is even. What properties does this relation satisfy? Is this an equivalence relation? What about if we change the condition to $a \sim b$ if $a + b$ is odd?

5. (A relation from linear algebra). Let's revisit that linear algebra part of your brain! For this problem, feel free to use results from your linear algebra textbook (e.g. Lay, ˜p.350) for this problem.

Let V be a subspace of $\mathbf{R}^n$ and let $W = V^\perp$ denote the orthogonal complement of V. Define $\sim$ on $\mathbf{R}^n$ by $x \sim y$ if and only if $x - y \in V$.

(a) Prove that $\sim$ is an equivalence relation on V.
(b) Prove that if $w, w' \in W$ and $w \neq w'$, then $w \not\sim w'$.
(c) Prove that if $x \in \mathbf{R}^n$, then there exists a unique $w \in W$ such that $x \sim w$.
(d) Let $r \in \mathbf{R}$ and $x, y, z, w \in \mathbf{R}^n$. Prove that if $x \sim y$ and $z \sim w$, then $(x + z) \sim (y + w)$ and $rx \sim ry$.

No new groupwork this week. Finish up the groupwork from HW 7.