MATH 3040 Prove It!
Fall 2017

Preliminary Exam #1

INSTRUCTIONS

- You have 50 minutes.
- The exam is closed book, closed notes, no calculators. However, you are allowed a one-page (front and back) "cheat sheet." **If you use such a sheet, submit it with your exam.** You are free to apply any result that we covered in class or on the homeworks, unless the problem explicitly tells you to use a certain approach. You do not need to cite the name and number of such results, just be clear on which result you are using.
- Mark your answers ON THE EXAM ITSELF (in particular, no exam books or loose sheets of paper). If you are not sure of your answer, you may wish to provide a *brief* explanation so that we can at least know what you are trying to do. For full credit, be sure to justify your steps.
- Write your name on the top of each page with a problem listed.
- Questions are not given in order of difficulty. Make sure to look ahead if stuck on a particular question.

Last Name	
First Name	
Student ID	
All the work on this exam is my own. **(please sign)**	

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Q. 1	Q. 2	Q. 3	Total
/20	/20	/20	/60

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1. (20 points) True/False Short Answers

If True, justify your answer with a proof. If false, give a counterexample (and show why it is indeed a counterexample). (You do not need to salvage false statements.)

(a) Let A and B be any two sets. We have $A \subset B$ if and only if $A \cup B = B$.

(b) If A and B are two sets such that $A \cup B \subseteq A \cap B$, then $A = B$.

(c) Let x be a real number. If x^5 is not a rational number, then x is not a rational number.

(d) If $f : A \to B$ and $g : B \to C$ are two functions between sets such that g is not injective, then their composition $g \circ f : A \to C$ is not injective.

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2. (20 points) Symmetric Difference.

Given any two sets A and B, the **symmetric difference of** A **and** B is defined as

$$A\Delta B = (A\backslash B) \cup (B\backslash A),$$

the union of the complement of B relative to A and the complement of A relative to B. (Recall that the complement of B relative to A consists of all the elements of A that are not also in B.)

(a) (10 points) Prove that $A\Delta B = \emptyset$ if and only if $A = B$. (Hint: for one direction, it may help to show that $A\Delta B = (A \cup B)\backslash(A \cap B)$.)

(b) (10 points) Prove that if A, B, C are sets, then $A \cap (B\Delta C) = (A \cap B)\Delta(A \cap C)$. (Hint: I recommend showing that the statements necessary for an element to be included in each set are logically equivalent.)

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3. (20 points) Induction. Prove the following statements by using some form of induction.

(a) (10 points) Define a sequence of numbers by $a_1 = 1, a_2 = 2, a_3 = 3$ and $a_n = a_{n-1} + a_{n-2} + a_{n-3}$ for all natural numbers $n \geq 4$. Then $a_n < 2^n$ for all $n \in \mathbf{N}$.

(b) (10 points) For all integers $n \geq 3$, we have

$$2 \cdot 3 + 3 \cdot 4 + \cdots + (n-1) \cdot n = \frac{(n-2)(n^2+2n+3)}{3}.$$

(The algebra for this last one can get a little tricky if you make the wrong move or proceed blindly, so be careful and organized.)

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