ANSWERS TO CLASS ASSIGNMENTS (WEEK OF 04/27)

Here are some solutions and commentary to the class assignments from the week of April 27. These are not the only way to do it, and they are not even "model solutions"—your solution may be better in a number of ways, and these might not necessary be "full credit" answers (especially with respect to the meta-mathematical questions)—but they give an idea of what we'd consider an acceptable answer to certain questions.

Planes, Proposition 7. At least three points on any line in a projective plane.

Proof. Given a projective plane $\Pi = (\mathcal{P}, \L)$, let $L \in \L$ be an arbitrary line. By (P1), L contains at least two points, call them A and B.

By (P4), there is at least one point Q that is not on L. By (P2), there exists a (unique) line L_A through A and Q and another line L_B through B and Q.

By (P4) again, there is another point R that is distinct from A, B, and Q and does not lie on any of the lines L, L_A, L_B. Consider the line L_R through Q and L, which exist by (P1). The line L_R must intersect our initial line L at a point S by (P3).

We need to show that S is not A or B. Suppose that for the sake of contradiction that $S = A$ (the case $S = B$ is the same). Then L_R contains the points A, Q, and R. But the unique line through A and Q is L_A, so this would imply that $L_R = L_A$, which contradicts the defining property of our point R.

Since $L_R \cap L = S$ and S is not A or B, we conclude that L contains at least three points: A, B, and S. $\square$

Remark. A common error that I saw was to first use (P4) and then use a line that runs through these four points. This is not correct, as it excludes all the lines that do not run through these four points. This can be fixed by first taking the line that you're considering, and *then* applying the axiom (P4) to guarantee the existence of your additional points. Much like mathematical statements themselves, the *order* of the sentences in the arguments affect the generality at which a proof applies.