MATH 3040: HOMEWORK 8

All answers must be in complete sentences. All assertions must be proven, per the student contract. Remember that this is a writing class, so while correctness and thoroughness are most important, *style* matters as well.

1. (Practice in Posing Questions) What is an interesting open-ended question you could ask, which is related to the material that we covered in class? What is a concrete mathematical question that could be used to approach an answer to the question?

2. (Prove or disprove and salvage) Suppose that we have functions $f : A \to B$ and $g : B \to C$, and define $h = g \circ f : A \to C$. Prove or disprove and salvage each of the following statements.

(a) If h is injective, then f is injective.
(b) If h is injective, then g is injective.
(c) If h is surjective, then f is surjective.
(d) If h is surjective, then g is surjective.

3. (Cardinality)

(a) Pick out 20 distinct numbers from the set

$$\{1, 4, 7, 10, 13, 16, 19, \ldots, 97, 100\}.$$

Prove that no matter how you make those choices, you will have chosen two numbers whose sum is 104.

(b) Create an injection from $\mathcal{P}(\mathbf{N})$ (the power set of the natural numbers) to the real numbers $\mathbf{R}$.

4. (Hyperovals) Let Π be a finite projective plane of order q. A **hyperoval** in Π is a set of $q + 2$ points, no three of which are collinear.

(a) A line is **tangent** to a hyperoval if it contains exactly one of its points. Prove that hyperovals do not have tangent lines. (Hint: Think about lines through a point of the hyperoval.)
(b) Show that if there exists a hyperoval in Π, then q is even. (Hint: Pick a point not on the hyperoval. Think about lines through this point.)

5. (Properties of Affine and Projective Planes)

(a) If L_1 and L_2 are two distinct lines in a projective plane Π, then there exists a point P in Π that does not lie on L_1 or L_2.

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(b) Show that on a finite affine plane, all lines have the same number of points lying on them.

The following question is to be done as a group.

6. (Groupwork) ("WLOG") (a) Consider the following game played by two players A and B, where the players take turns and A goes first. You place four points (labelled 1, 2, 3, 4) on a chalkboard such that no three of the points lie on the same straight line. On each player's turn, they must pick two of the four points that have not been connected and draw a straight line segment between them. However, a player is not allowed to form a triangle whose vertices are among the original four starting points. (If you accidentally form a triangle by criss-crossing lines, but where the vertices of that triangle are not all among the four starting points, that is fine.) The first player unable to move loses.

Which player has a winning strategy? (Hint: To simplify things, you must understand how certain positions are "essentially the same"; for example, all moves by A are essentially the same, so one can assume that the move connects points 1 and 2. Then B has essentially two distinct choices: draw a segment adjacent to the A's segment or not.)

(b) What if you have the same setup, but where your goal is to *create* the first triangle. Which player has a winning strategy?