MATH 3040: PROOF PORTFOLIO

Learning to write mathematical proofs takes time and a lot of practice. The best way to improve your proof-writing skills is to try to write a proof, receive feedback on your work, and then *rewrite your proofs* based on that feedback. The goal of the proof portfolio is to have you write several drafts of proofs throughout the semester. You want to build a portfolio of well-written proofs by the end of the semester.

This is a major project, and should take most of the semester to complete. It will represent a significant portion of your grade when you turn in the final version at the end of the class. The drafts will not be graded (except for completeness), but will be an opportunity to get feedback.

You can include anything you would like in your proof portfolio: proofs that you worked out from the sheets in class, proofs that you did on homework assignments, proofs from your own mathematical investigations that you pursued, etc.; anything that can be related to material from the class. However, they should represent your best work, so choose them carefully. Everything should be in your own words, be in complete sentences, and be free of spelling and grammar errors.

For your first draft submissions, you can use any TeX template you like. Your drafts should be submitted to Gradescope by the appropriate date and time.

For your second draft, use the feedback on the first draft to pick and choose your proofs and refine their style.

Notes on content:

- In selecting questions, *originality* counts. For example, your proofs should not just be your answers to assigned homework questions. There should be at least a couple that you asked and answered on your own, such as your answers to questions to the first question on each homework, those you came up with to review for the exams, or that you explored on your own.
- In selecting proofs to present, make sure they represent a variety of skills. For example, they shouldn't all be induction proofs or all proofs involving long algebraic manipulations, all proofs by contradiction, all proofs using geometric arguments, all proofs involving counting, all short proofs that you can prove without intermediate lemmas, etc.
- Your proofs to propositions on the in-class sheets are a good source if you need short proofs that demonstrate your facility with a new

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concept. Finding interesting examples and showing that they satisfy certain properties is also a good way to do this.

• Avoid chalkboard shorthand in writing your proofs. For example, you shouldn't use symbols like " \Rightarrow " except in logic questions, or abbreviations like RHS, LHS, WLOG, etc.

Notes on style:

- Decide what's important to say. Writing well doesn't necessarily mean writing more. If your solution is too wordy, it can sometimes obscure your point. Your primary role is to give insight.
- Signpost. If you have a long or complicated argument, help the reader by giving an outline of your approach in the beginning.
- Omit details that have no bearing on the problem. For example, if you write "we express a rational number r as m/n where m and n have no common factors," that indicates that you'll use the "no common factors" property later. If you don't use it, leave out the extraneous fact and write it differently.
- Step back and simplify. After you've written down a proof, ask yourself: can this argument be streamlined? A very common way this occurs is if you do a proof by contradiction, because in many cases (though not always!) a direct proof is shorter. A good way to find these is to try and prove an argument in multiple ways (e.g. direct proof vs. contrapositive, using a geometric argument vs. finding a formula, etc).

For your final draft:

- You have *fewer* pages in the final draft than the second, so make sure that the final draft represents your best polished work. Not filling out all the pages or having "filler" in the problem or proofs is a bad sign.
- Be *doubly sure* that your proofs are correct, your logic is sound, and that you're not, say, missing any cases in your proofs. We have not really focused on this in the draft reviews, but it will be scrutinized in the final draft.
- As we've emphasized throughout the class, *style matters*. Make sure you're writing in complete sentences, that you have transitions between arguments, etc. It should be easy to read and it should be clear at all times what part of the problem you are addressing. An important part of your grade for the proof portfolio will be on the style of your statements and proofs.
- Check for proper spelling and grammar and fix all formatting errors. The content is of primary importance, of course, but having this demonstrates that you have paid close attention to the details. Small mistakes like these indicates a lack of attention and are often a place where errors occur.

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Important dates:

- Wednesday, October 11, 2017: Submit your first draft (10 pages).
- Wednesday, November 15, 2017: Submit your second draft (20 pages). There should be a longer proof among your selection, such as one that requires you to prove a couple of intermediate lemmas along the way.
- Friday, December 1, 2017: Final draft due (15 pages + 2 page introduction). Include a 1–2 page introduction that explains why you have chosen the proofs and what qualities of yours are highlighted in each proof. Here's an example: "The statement and proof in #5 is a question that arose after a class session, and which I pursued in a discussion with my group members that week. Aside from demonstrating my facility with the course content on projective planes, it applies a form of induction that requires two base cases and a delicate counting argument. I also thought it was awesome that I could ask and answering a cool question using just the tools from class!" Do this for each proof you include.