MATH 3110: Homework #2
(due Tuesday, September 6)

August 30, 2016

The references to [Mattuck] refer to the **Exercises** at the end of chapter (p. 12 in my version). If you'd like to receive full credit, be sure to show your reasoning. Try to write in complete sentences.

1. Prove that if n is a natural number $(1, 2, 3, 4, \ldots)$ other than 3, then $2^n \geq n^2$. (Hint: it might help to show first that if $n \geq 3$, then $2n^2 \geq (n+1)^2$.)

2. [Mattuck 1.6/4] Prove the sequence

$$a_n = \frac{1 \cdot 3 \cdots (2n-1)}{2 \cdot 4 \cdots 2n}$$

has a limit.

3. [Mattuck 2.1/3] "If $\{a_n\}$ and $\{b_n\}$ are increasing, then $\{a_n b_n\}$ is increasing." Show this is false, change the hypothesis on $\{b_n\}$ and prove the amended statement.

4. [Mattuck 2.2/2] Give an upper estimate for $\ln 3 = \int_1^3 \frac{dx}{x}$ as in example 2.2C, by using (a) one trapezoid; (b) two trapezoids.

5. [Mattuck 2.4/2] If $|a_1 \sin b + a_2 \sin 2b + \cdots + a_n \sin nb| > n$, prove that $|a_i| > 1$ for at least one of the a_i. (Use contraposition: cf. Appendix A.2.)

6. [Mattuck Problem 2-1] Let $\{a_n\}$ be a sequence. We construct from it another sequence $\{b_n\}$, its *sequence of averages*, defined by

$$b_n = \frac{a_1 + \cdots + a_n}{n} = \text{ average of the first } n \text{ terms.}$$

(a) Prove that if $\{a_n\}$ is increasing, then $\{b_n\}$ is also increasing.

(b) Prove that if $\{a_n\}$ is bounded above, then $\{b_n\}$ is bounded above.