MATH 3110: Homework #7
(due Tuesday, October 18)

October 11, 2016

The references to [**Mattuck**] refer to the **Exercises** at the end of chapter. If you'd like to receive full credit, be sure to show your reasoning. Try to write in complete sentences.

1. [**Mattuck Problem 11-2**] Call a function "multiplicatively periodic" if there is a positive number $c \neq 1$ such that $f(cx) = f(x)$ for all $x \in \mathbf{R}$. Prove that if a multiplicatively periodic function is continuous at 0, then it is a constant function.

2. [**Mattuck 12.1/1, 12.1/3**] (12.1/1) Prove that a continuous function whose values are always rational numbers is a constant function.

(12.1/3) Give an example of a function with two zeros, one of which cannot be found by the bisection method associated with Bolzano's Theorem. Can Newton's method find this zero?

3. [**Mattuck 12.2/2**] (a) Approximately where are the roots of $x^4 - \epsilon x^2 - 1 = 0$ if ϵ is a small posiive number? (Do not use the quadratic formula.)

(b) Let r be the highest root; r depends on ϵ, that is, $r = r(\epsilon)$. Show that $\lim_{\epsilon \to 0} r(\epsilon)$ exists and determine its value.

(Follow the similar examples in the text for both parts. The type of reasoning in part (a) is an example of what are called *perturbation methods*: varying the parameters in a problem slightly, and studying how this affects the solutions.)

4. [**Mattuck Problem 12-5**] **Inscribing equilateral triangles.** Let C be a smooth, convex, closed curve, i.e., one without endpoints, and such that a

line segment joining any two points on C lies inside C. (An ellipse or an oval are examples. "Smooth" means it has a tangent line at each point.)

Let P be any point on C. Show convincingly that you can always find two other points Q and R on C such that PQR is an equilateral triangle. (Try some sketches.)

5. [**Mattuck Problem 12-7**] **Danish ham sandwich.** A Danish open-faced ham sandwich consists of a thin slice of rye bread and a thin slice of ham. Both have irregular shapes, but assume they have uniform thickness. Prove it is possible with a single vertical knife-cut to divide any ham sandwich in two so that each person gets half the ham and half the bread.

6. [**Mattuck 13.4/1, 2**] (13.4/1) Let $f(x)$ be a function defined for all x that maps compact intervals to compact intervals: *I compact interval* $\Rightarrow$ *$f(I)$ compact interval.* Do *not* assume $f(x)$ is continuous.

(a) Prove that on any compact interval I the function attains its maximum and minimum (as in the conclusion of the Maximum Theorem).

(b) Prove that $f(x)$ has the *Intermediate Value Property*: for any compact interval $[a, b]$, $f(x)$ takes on all values between $f(a)$ and $f(b)$ as x varies over the interval.

> This exercise shows the Maximum Theorem and Intermediate Value Theorem follows from the Continuous Mapping Theorem, and therefore taken together they are equivalent to the Continuous Mapping Theorem.

(13.4/2) Give an example of a function defined for all x which is not continuous, yet which satisfies the italicized property on line 2 of the previous exercise (that is, the Intermediate Value Property).