MATH 3110: Homework #8
(due Tuesday, October 25)

October 18, 2016

The references to [**Mattuck**] refer to the **Exercises** at the end of chapter. If you'd like to receive full credit, be sure to show your reasoning. Try to write in complete sentences.

1. [**Mattuck 13.1/1**] (a) Show that if S is the union (cf. A.0) of a finite number of compact intervals, then S is sequentially compact. (The intervals need not overlap.)

(b) Show by counterexample that this is no longer true if S is the union of an infinite number of compact intervals.

2. [**Mattuck 13.2/2**] Using the ideas of this section (i.e., without using derivatives), prove that a polynomial of even degree either has a maximum or a minimum on $(-\infty, \infty)$. Give a simple criterion for deciding which it has.

3. [**Mattuck 13.5/1,2,4**] (13.5/1) Prove directly from the definition of uniform continuity that $\sin x$ is uniformly continuous on $(-\infty, \infty)$. (The unit circle picture helps to make the estimations needed; see Example 11.1B.)

(13.5/2) Prove that a function which is continuous and periodic on $\mathbf{R}$ is uniformly continuous on $\mathbf{R}$. (This gives an alternative proof for Exercise 13.5/1.)

(13.5/4) Is $\sqrt{x}$ uniformly continuous on $[0, \infty)$? Justify your answer. (Note that its slope at 0 is infinite. There are several approaches to this exercise; one of them uses the preceding exercise.)

4. [**Mattuck Problem 13-1**] Suppose $f(x)$ is continuous on some open interval I, and c is a maximum point for $f(x)$ inside this interval.

Common sense suggests that $f(x)$ should be increasing immediately to the left of c and decreasing immediately to the right of c. Is this true? Either prove it, or give a counterexample. (Note that a constant function is considered to be both increasing and decreasing.)

5. [**Mattuck Problem 13-3**] Prove directly from the definition of uniform continuity that a polynomial $f(x)$ is uniformly continuous on a compact interval.

(Hint: $f(x') - f(x'') = (x' - x'') \cdot g(x', x'')$, where g is a polynomial in two variables; prove this, and then use it. Note that at this point we do not have any definitions or theorems about functions of two variables.)

6. [**Mattuck Problem 13-4**] Let $f(x)$ be continuous on the compact interval I, and suppose that $f(x)$ has an infinity of maximum points (i.e. points where it takes on its maximum value on I), an infinity of minimum points on I, and that between any two maximum ppoints lies at least one minimum point.

Prove $f(x)$ is constant on I. Why isn't $\sin(1/x)$ a counterexample?