HW #6 solutions

1. (KW 4.23) $\delta'_{ij} = \delta_{rs} t_{ir} t_{js} = \delta_{ij}$

δ_{ij} only depends on the indices

consider the dot product between 2 vectors, one in a right-handed system; the other in a (primed) left-handed system.

$A_i B_j \delta_{ij} = A'_r B'_s \delta'_{rs}$ (invariance of dot product)

now, we re-express RHS as $a_{ri} A_i a_{sj} B_j \delta'_{rs}$

true for arbitrary A_i, B_j

so $\delta_{ij} = a_{ri} a_{sj} \delta'_{rs}$ → regular tensor transformation

2. (KW 5.2 (v))

$\int_0^{2\pi} \delta(\cos(t))\,dt$ recall $\delta(f(t)) = \sum_{t_i : f(t_i)=0} \left|\left| \frac{1}{\frac{\partial f}{\partial t}\big|_{t_i}} \right|\right| \delta(t - t_i)$

$\cos(t)$ hits 0 twice in the interval: at $\frac{\pi}{2}$ and at $\frac{3\pi}{2}$

$$\delta(\cos(t)) = \left| \frac{1}{-\sin(t)\big|_{\pi/2}} \right| \delta\left(t - \frac{\pi}{2}\right) + \left| \frac{1}{-\sin(t)\big|_{\frac{3\pi}{2}}} \right| \delta\left(t - \frac{3\pi}{2}\right)$$

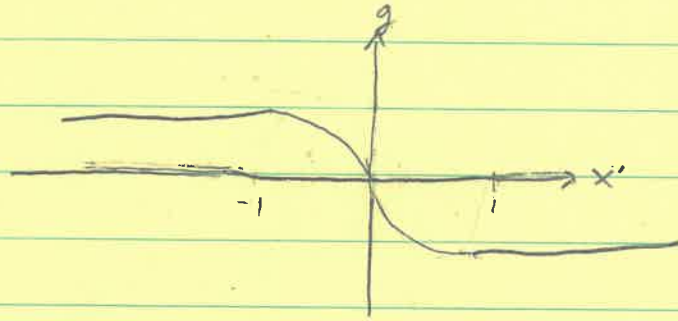
$$= \delta\left(t - \frac{\pi}{2}\right) + \delta\left(t - \frac{3\pi}{2}\right)$$

so $\int_0^{2\pi} \delta(\cos(t))\,dt = 1 + 1 = 2$

3. (KW 5.4)

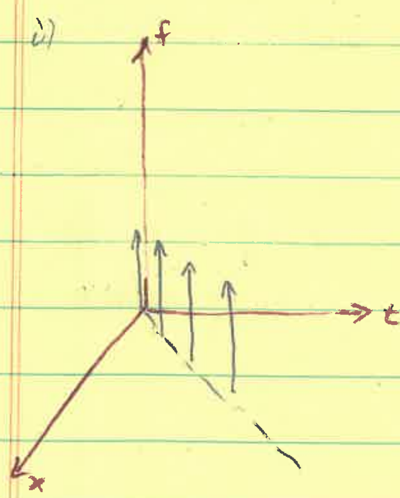
$$h(x) = \int_{-\infty}^{\infty} dx'\, g(x') \frac{d}{dx'} \delta(x - x') = - \frac{\partial g(x')}{\partial x'}\Big|_x$$

so
$$g(x') = \begin{cases} ct. & x < -1 \\ -\frac{x'^2}{2} - x' & -1 < x' < 0 \\ \frac{x'^2}{2} - x' & 0 < x' < 1 \\ ct. & 1 < x \end{cases}$$



4. (KW 5.6)

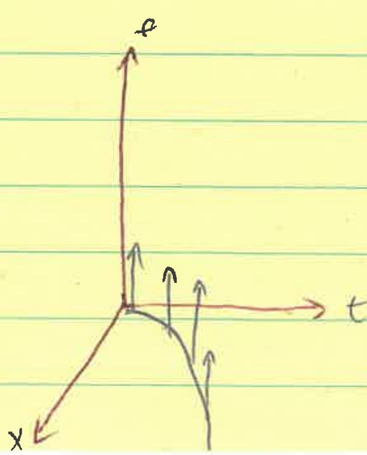
i)



arrows in pencil are δ - functions

dotted line is $x = v_0 t$

ii



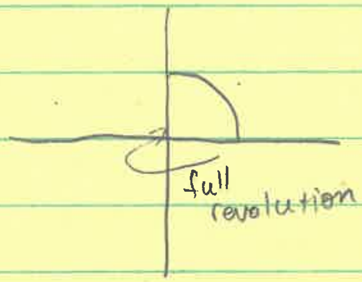
δ - function moves along parabola

$x = \frac{a_0}{2} t^2$

a_0 has units of acceleration

5. Assume $\boxed{\rho = AQ_0\,\delta(r-r_0)\cos(\theta)}$. consider upper hemisphere.

$$Q_0 \overset{?}{=} \underbrace{\int_0^\infty r^2\,\delta(r-r_0)\,dr}_{r_0^2}\;\underbrace{\int_0^{2\pi} d\varphi}_{2\pi}\;\underbrace{\int_0^{\pi/2}\cos(\theta)\sin(\theta)\,d\theta}_{\frac{1}{2}} \cdot AQ_0$$



$$Q_0 = \pi r_0^2\,Q_0 A$$

$$\therefore \quad A = \frac{1}{\pi r_0^2}$$

$$\boxed{\rho_\pm = \frac{\pm Q_0}{\pi r_0^2}\cos(\theta)\,\delta(r-r_0)}$$

where $\pm$ refers to the positive or negative hemisphere